Karlsruher Institut für Technologie

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Necessary and Sufficient Conditions for the Existence of a Product Structure

Projektgruppe "Praxis der Forschung"
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1 Motivation

Product structure is a novel technique in structural graph theory to represent all graphs of a given class as subgraphs of the strong graph product of a graph of constant treewidth and a path. It has been recently introduced to obtain several breakthrough results in graph coloring and graph representation problems. Among others, it is known that the class of planar graphs admits product structure with the treewidth constant between 3 and 6. While for some graph classes product structure is known not to exist, it remains an interesting open problem ffor several other graph classes.

2 Project

This project is concerned with necessary conditions and sufficient conditions for a graph class to admit product structure. In particular, it is necessary that the subgraphs induced by the neighborhood of a single vertex have constant treewidth. However, this very local condition is not sufficient as there can already be obstructions just "one level deeper". Within this project we seek to find a set of necessary conditions based on the subgraphs of all iterated neighborhoods, and determine whether or not the entirety of all these conditions is already sufficient for product structure, or whether there are further, more global obstructions. As concrete graph classes we shall consider intersection graphs of geometric objects in low dimensions, such as disks and axis-aligned squares in the plane, where in order to meet the first local condition, we restrict to those graphs of constant clique number.

The students in this project shall first review the literature on product structure and closely related concepts in structural graph theory, such as treewidth and pathwidth. Having a complete understanding of the current state of the art, the original investigations shall start with intersection graphs of disks in which each disk has an unobstructed center, or with low ply value, or in which each disk has a constant fraction of unobstructed area. In such scenarios we seek to understand which large treewidth graph of low clique number can exist in the neighborhood of a single vertex.

3 Contact

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