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Algorithmik I
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Algorithms for Computing Electrical Flows

Background.

A *flow* in a directed graph $G = (V, E)$ with vertex demand function $d\colon V \to \mathbb{R}$ is a function $f\colon E \to \mathbb{R}$ such that the flow is conserved at all vertices. That is, at each vertex the sum of flows on the incident edges equals the demand of the vertex. Formally, for every vertex $v \in V$ we have

$$\sum_{u\in V\colon uv\in E} f(uv) - \sum_{w\in V\colon vw\in E} f(vw) = d(v). \tag{1}$$

This property is called the *conservation of flow* or *Kirchhoff's current law*.

In the context of electrical flows each edge $e \in E$ has a positive resistance $r(e)$. The *voltage* of an edge e is $f(e) \cdot r(e)$. An *electrical flow* is a flow that satisfies *Kirchhoff's voltage law*, which states that the voltages along the edges of any cycle sum to 0, i.e., for every cycle C we have

$$\sum_{e\in C} r(e) \cdot f(e) = 0. \tag{2}$$

Equivalently, Kirchhoff's voltage law can be formulated in terms of vertex potentials $\theta\colon V \to \mathbb{R}$. A flow f is an electrical flow if and only if there are vertex potentials θ such that $r(vw) \cdot f(vw) = \theta(w) - \theta(v)$ for every edge $vw \in E$.

Directions.

Include but are not limited to:

- Develop algorithms to compute electrical flows faster than existing algorithms.
- Further explore the structure of electrical flows.
- Study problems related to electrical flows, e.g., transmission network design problems and effective resistances.

All three directions require first a study of the existing literature, including a thorough understanding of existing techniques. Afterwards, there are several possible routes to take: From purely theoretical original research with structural or algorithmic considerations to the implementation and comparison of existing and/or new algorithms.

Requirements.

Students should be familiar with flow networks, linear algebra, combinatorial reasoning, and basic computational aspects of theoretical computer science. More importantly, one should be interested in learning classic and recent techniques for computing electrical flows but also willing to dive into mostly unexplored concepts. This endeavor requires curious and creative students that are not afraid to leave the beaten path.