Formal Specification of Software

Bernhard Beckert

Adaptation of slides by
Wolfgang Ahrendt
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in the object-oriented setting:
The units to be specified are interfaces, classes, and their methods.

We first focus on specifying methods.

Methods are specified by potentially referring to:

- the result value,
- the initial values of formal parameters,

But what do we mean by state?
Unit Specifications

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But what do we mean by state?
Prerequisite: Object-oriented States

By state, we mean a ‘snapshot’ of the system, at any point during the computation, described in terms of the programmer’s model.

An object oriented state consists of:

- the set $C$ of all loaded classes
- the values of the static fields of classes in $C$
- the set $O$ of references to all created objects
- the values of the instance fields of objects in $O$

Here, values of local variables and formal parameters are not considered part of the state.
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Here, values of local variables and formal parameters are not considered part of the state.
Like implementations, specifications can only refer to the locally visible part of the state (e.g., not to private fields of other classes).
Prerequisite: Visible State

In our context, we stick to the following principle:

**Same Visible State for Specifications and Implementations:**

In some local context, specifications and implementations can access the same part of the overall state.\(^a\)

\(^a\)Later, we’ll refine this principle, and introduce well defined exceptions.

Thus, specifications talk only about those parts of the state which are accessible by:

- respecting JAVA’s visibility rules (public, protected, private),
- following (visible) references, starting from local fields.
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A purely functional specification of a (non-void) method talks
- only about
  - the result of a call
  - the initial value of input parameters
- but not about
  - (any part of) the state

examples:

**interface/class:** Math  
**method:**
- static int abs(int a)
- static double sqrt(double a)
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from the Java API:

**Specification of static int abs(int a)**

Returns the absolute value of an int value. If the argument is not negative, the argument is returned. If the argument is negative, the negation of the argument is returned.

Note that if the argument is equal to the value of `Integer.MIN_VALUE`, the most negative representable int value, the result is that same value, which is negative.

**Green**: Intuitive description rather than a specification.

**Red**: Precise specification by case distinction, given we know what ‘negative’ and ‘negation’ mean exactly.

**Blue**: A consequence of the specification, i.e. a redundant part of it.

**Red** and **Blue** are candidates for formalisation.
Specifying the `Math::abs()` function from the Java API:

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Going a bit more formal

static int abs(int a)

Informal spec:
If the argument is not negative, the argument is returned. If the argument is negative, the negation of the argument is returned.

Semi formal:
- Under the precondition ‘a ∈ [0...2147483647]’, abs ensures the postcondition ‘result = a’.
- Under the precondition ‘a ∈ [−2147483648... − 1]’, abs ensures the postcondition ‘result = −a’.
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Note that if the argument is equal to the value of `Integer.MIN_VALUE`, the most negative representable int value, the result is that same value, which is negative.

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- Under the precondition 'a = −2147483648', `abs` ensures the postcondition 'result = −2147483648'.

*Or simply:*\(^1\)
- `abs(−2147483648) = −2147483648`

\(^1\)But be careful when using a method call in a formula, see below.
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A state aware specification of a (void or non-void) method talks about
- the result of a call (if non-void)
- the initial value of input parameters
- two states:
  - the ‘pre-state’ of the method call
  - the ‘post-state’ of the method call

examples:

\[\text{interface/class: } \text{method:}\]

List \hspace{1cm} Object set(int index, Object element)
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from the Java API of List::set (simplified):

public Object set(int index, Object element)

Replaces the element at the specified position in this list with the specified element.

**Parameters:**
index - index of element to replace.
element - element to be stored at the specified position.

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the element previously at the specified position.

**Throws:**
IndexOutOfBoundsException
- if the index is out of range (index < 0 || index >= size()).

Why is the spec state aware?
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set ensures the following postcondition:

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Reflection

We identify elements of a framework for *Formal Specification*

- pairs of
  - preconditions
  - corresponding postconditions
- a language to express these conditions, capturing:
  - relations, equality, logical connectives
  - *quantification*
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Consider Class SortedIntegers

```java
public class SortedIntegers {

    private int arr[];
    private int capacity, size = 0;

    public SortedIntegers(int capacity) {
        this.capacity = capacity;
        this.arr = new int[capacity];
    }

    public void add(int elem) { /* ... */ }

    public boolean remove(int elem) { /* ... */ }

    public int max() { /* ... */ }
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Which methods have purely functional / state aware specifications?
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Specification of int max()

max() returns the maximum of the elements in the array arr.

But that is not what we wanted.

max() should return the maximum of the elements which were already added, and not removed thereafter.
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`max()` returns the maximum of those elements in the array `arr` which were already added, and not removed thereafter.

How can we state this without referring to the history of the object?

We can use the fact that the integers are (supposed to be) sorted.
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Specification of `int max()` now much simpler

`max()` returns `arr(size-1)`.

Sufficient if we assume sortedness.

Questions:

A) how to express the sortedness property?
B) how to specify that an instance of `SortedIntegers` always has this property?
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A SortedIntegers object is sorted if:

for all $i \in [0...\text{size}() - 2]$: $\text{arr}(i) \leq \text{arr}(i+1)$

Below, we abbreviate this condition by ‘SORTED’.

Note:
Even SortedIntegers objects with with $\text{size}() \leq 1$ satisfy $\text{SORTED}$. 
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Even SortedIntegers objects with \( \text{size()} \leq 1 \) satisfy SORTED.
B) Specifying Sortedness

How to specify that sortedness is a property of a SortedIntegers object at any time?

State that \textit{SORTED} is invariant w.r.t. actions on SortedIntegers. i.e., \textit{SORTED} is:

- established by all constructors
- maintained by all methods
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**add SORTED to**

- postcondition of all constructors
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Problem: This way,
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Solution: Class Invariants

Invariant conditions belong to the *object*, not to the actions on object.

Attach invariant conditions to the *class*, not to methods/constructors. We call these conditions ‘class invariants’.

Constructors/methods of a class are *implicitly* (but firmly!) obliged to establish/maintain invariant conditions of their class.
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Specification Conditions

in summary: three types of conditions in specifications

- **preconditions** of methods
- **postconditions** of methods and constructors
- **class invariants**\(^2\)

\(^2\)not to be confused with loop invariants, see last part of course
We will use the ‘Java Modelling Language’ (JML) to specify Java programs.

JML combines

- Java
- First-Order Logic (FOL)

We first introduce First-Order Logic, and JML afterwards.
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- a set \( T_\Sigma \) of types
- a set \( F_\Sigma \) of function symbols, each with fixed typing
- a set \( P_\Sigma \) of predicate symbols, each with fixed typing
- a typing \( \alpha_\Sigma \)

The typing \( \alpha_\Sigma \) assigns
- to each function and predicate symbol:
  - its number of arguments (\( \geq 0 \))
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- to each function symbol its result type.

We assume set \( V \) of variables (\( V \cap (F_\Sigma \cup P_\Sigma) = \emptyset \)), each having a unique type.
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terms are defined recursively:

**Terms**

A first-order term of type $\tau \in T_\Sigma$

- is either a variable of type $\tau$, or
- has the form $f(t_1, \ldots, t_n)$,
  where $f \in F_\Sigma$ has result type $\tau$, and each $t_i$ is term of the correct type, following the typing $\alpha_\Sigma$ of $f$. 
Logical Atoms

A logical atom has either of the forms

- `true`
- `false`
- `t_1 = t_n` ("equality")
- `p(t_1, \ldots, t_n)` ("predicate"),
  where `p \in P_\Sigma`, and each `t_i` is term of the correct type, following the typing `\alpha_\Sigma` of `p`. 
first-order formulae are defined recursively:

**Formulae**

- each atomic formula is a formula
- if $\phi$ and $\psi$ are formulae, and $x$ is a variable, then the following are also formulae:
  - $\neg \phi$ ("not $\phi$")
  - $\phi \land \psi$ ("$\phi$ and $\psi$")
  - $\phi \lor \psi$ ("$\phi$ or $\psi$")
  - $\phi \rightarrow \psi$ ("$\phi$ implies $\psi$")
  - $\phi \leftrightarrow \psi$ ("$\phi$ is equivalent to $\psi$")
  - $\forall t \ x. \ \phi$ ("for all $x$ of type $t$ holds $\phi$")
  - $\exists t \ x. \ \phi$ ("there exists an $x$ of type $t$ such that $\phi$")
... we now would rigorously define:

- validity of formulae
- provability of formulae (in various calculi)

⇒ see course ‘Logic in Computer Science’

In our course, we stick to the intuitive meaning of formulae.

But we mention ‘models’.
Models vs. States

**Model**

A model assigns *meaning* to the symbols in $F_\Sigma \cup P_\Sigma$ (assigning functions to function symbols, relations to predicate symbols).

In a *given model* $M$, a formula is either *valid* or *not valid*.

**Tautologies**

A formula is a *tautology* if it is valid in all models.

In the context of formal specification of imperative programs: states take over the role of models.
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A model assigns *meaning* to the symbols in $F_\Sigma \cup P_\Sigma$ (assigning functions to function symbols, relations to predicate symbols).

In a **given model** $M$, a formula is either **valid** or not **valid**.

Tautologies

A formula is a **tautology** if it is valid in **all models**.

In the context of formal specification of imperative programs: **states**\(^3\) take over the role of **models**.

\(^3\)together with input values and results, and possibly paired with an old states
useful tautologies: whiteboard
We will use the ‘Java Modelling Language’ (JML) to specify Java programs.

### JML combines
- First-Order Logic (FOL)
- Java