Introduction to Artificial Intelligence

Logical Agents

(Logic, Deduction, Knowledge Representation)

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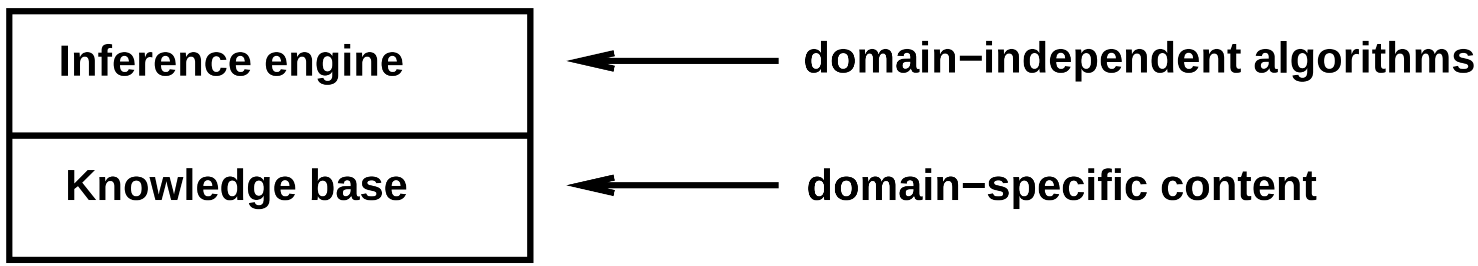
Outline

- Knowledge-based agents
- Wumpus world
- Logic in general—models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - forward chaining
  - backward chaining
  - resolution

Knowledge bases



Knowledge base

Set of sentences in a formal language

Declarative approach to building an agent

Tell it what it needs to know

Then it can ask itself what to do—answers follow from the knowledge base

Wumpus World PEAS description

Performance measure

gold +1000,   death -1000

-1 per step,   -10 for using the arrow

Environment

Squares adjacent to wumpus are smelly

Squares adjacent to pit are breezy

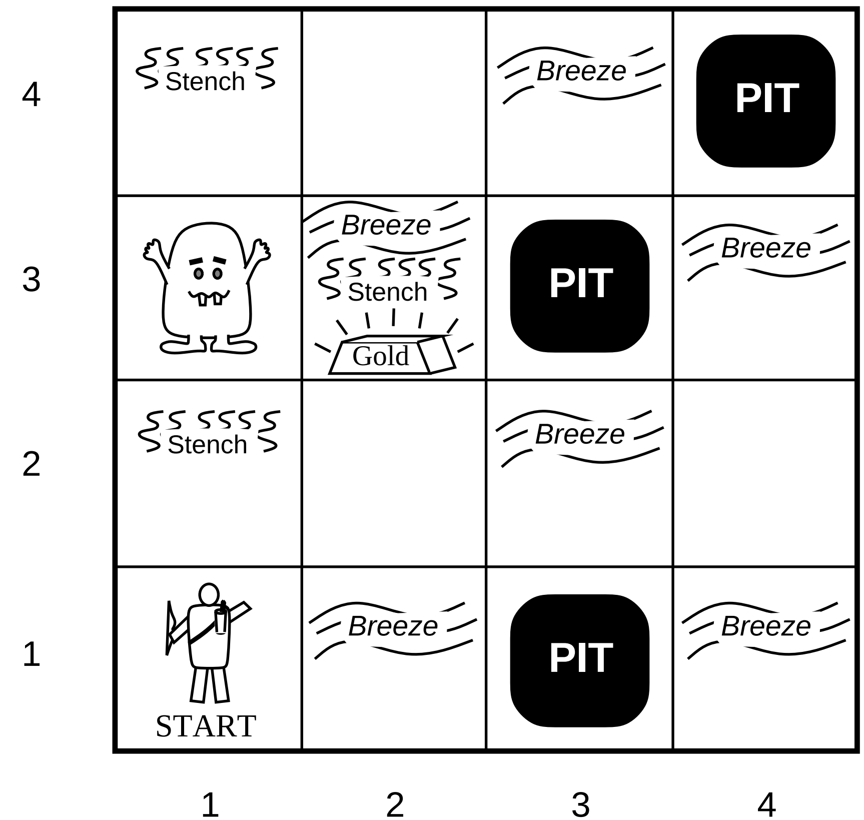
Glitter iff gold is in the same square

Shooting kills wumpus if you are facing it

Shooting uses up the only arrow

Grabbing picks up gold if in same square

Releasing drops the gold in same square



Actuators

Left turn, Right turn,

Forward, Grab, Release, Shoot

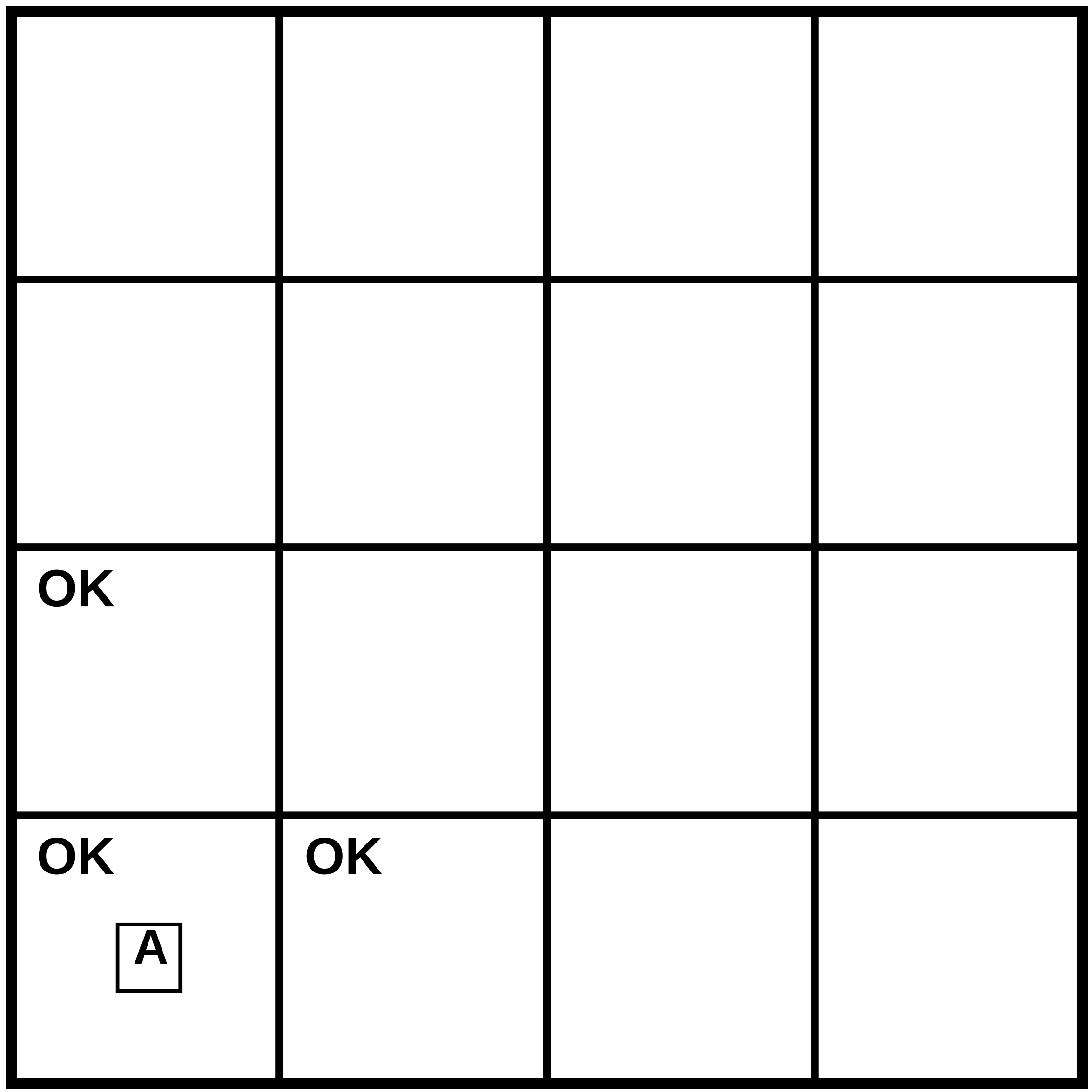
Sensors

Breeze, Glitter, Smell

Wumpus World Characterization

Observable	**No – only local perception**
Deterministic	**Yes – outcome of action exactly specified**
Episodic	**No – sequential at the level of actions**
Static	**Yes – wumpus and pits do not move**
Discrete	**Yes**
Single agent	**Yes – wumpus is essentially a natural feature**

Exploring a Wumpus World



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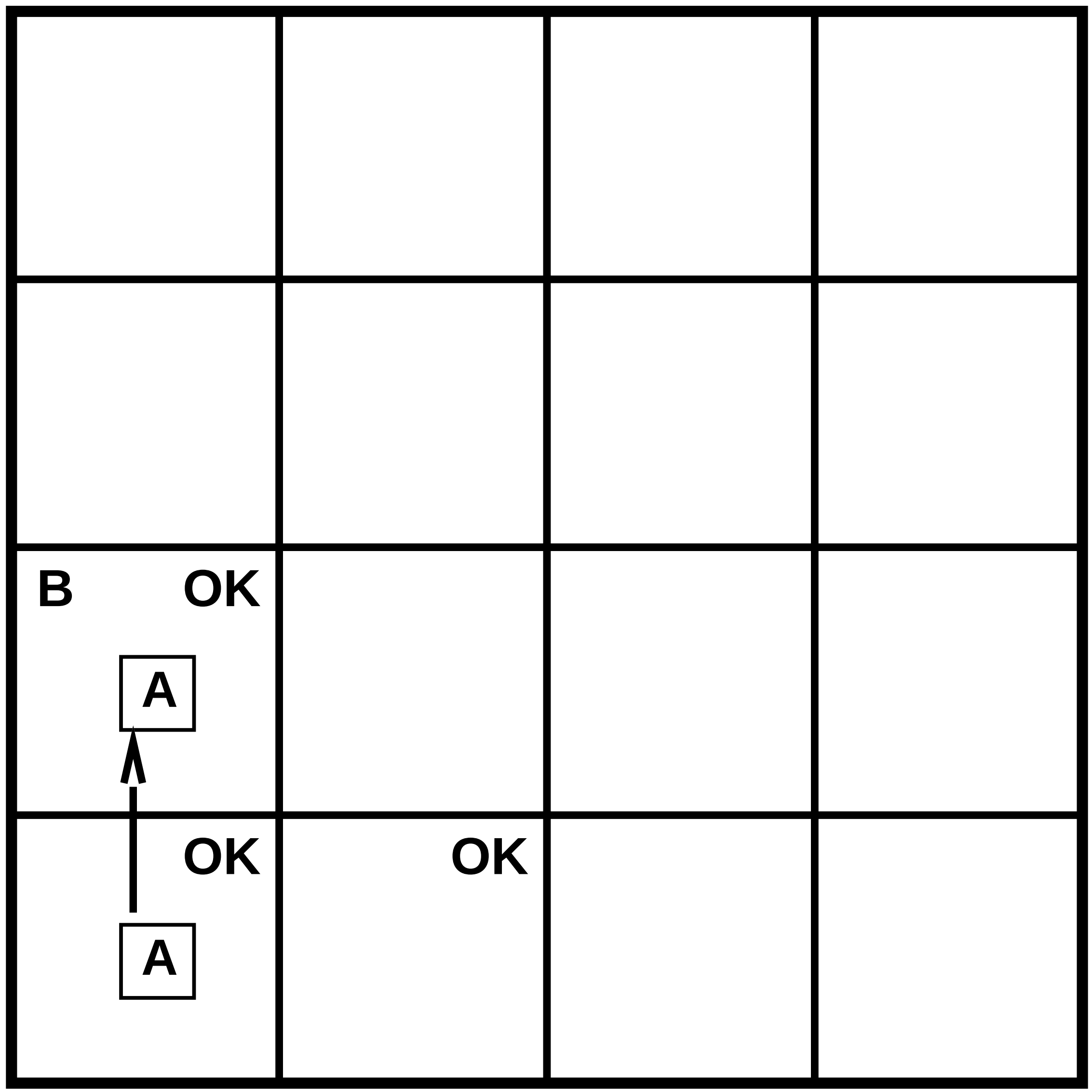

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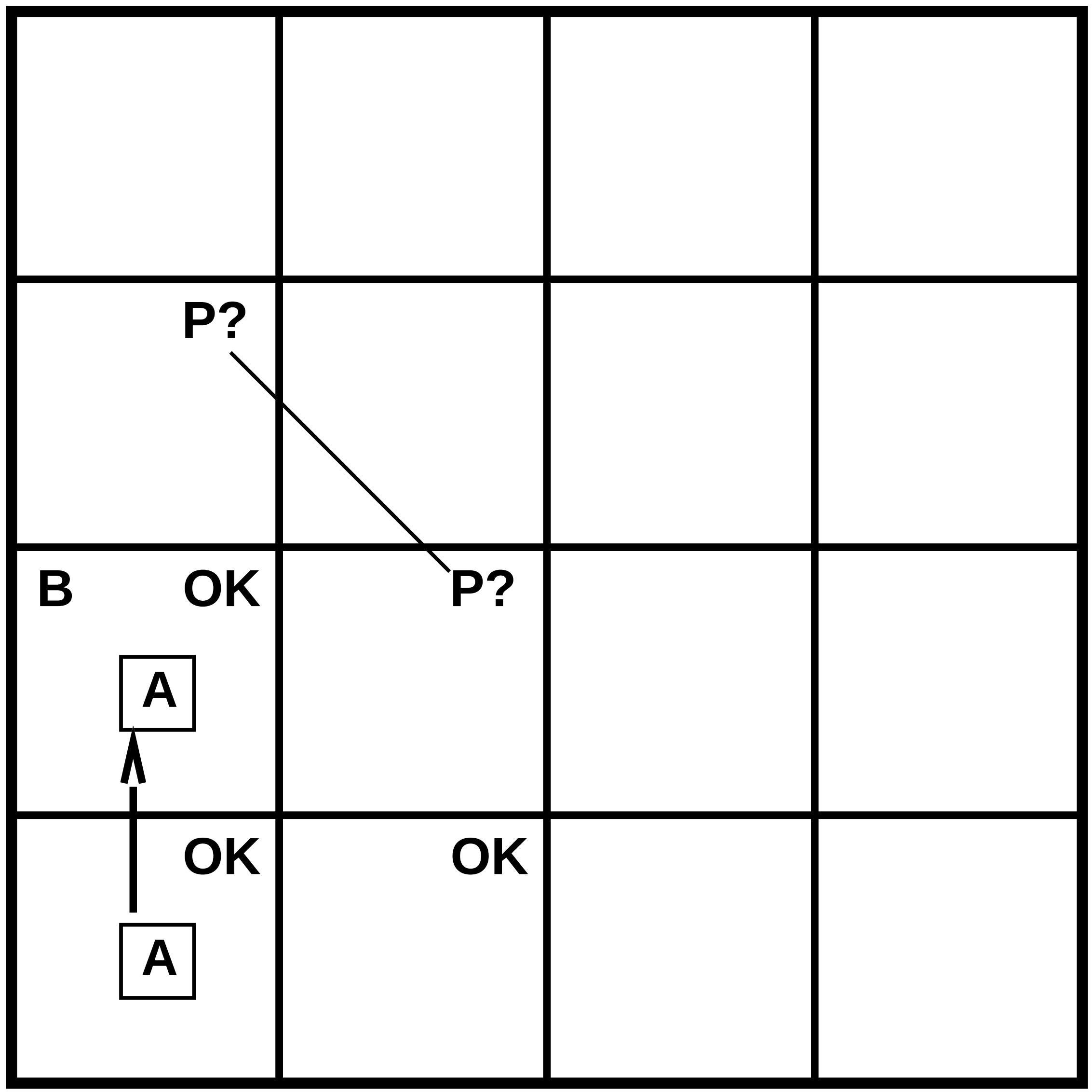

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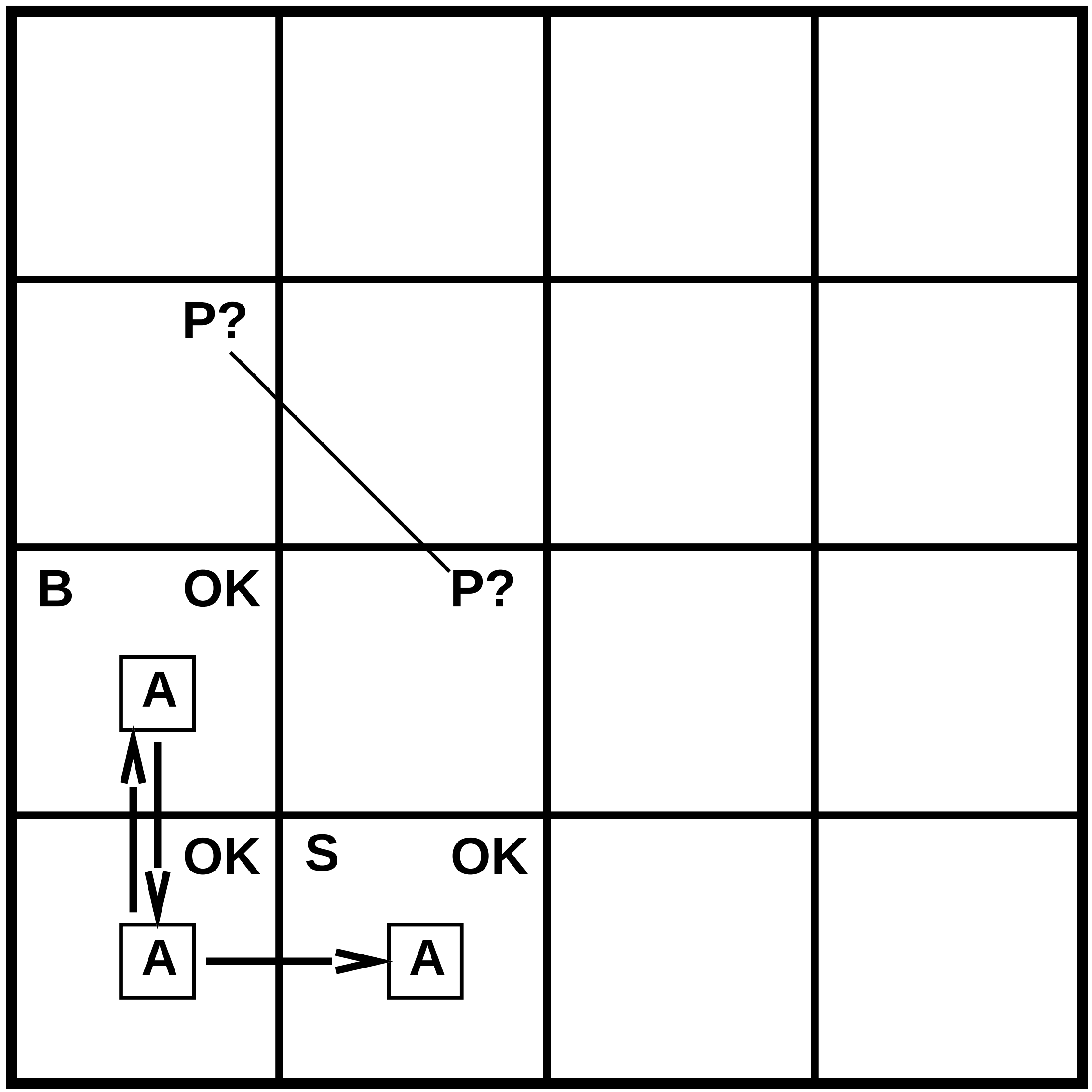

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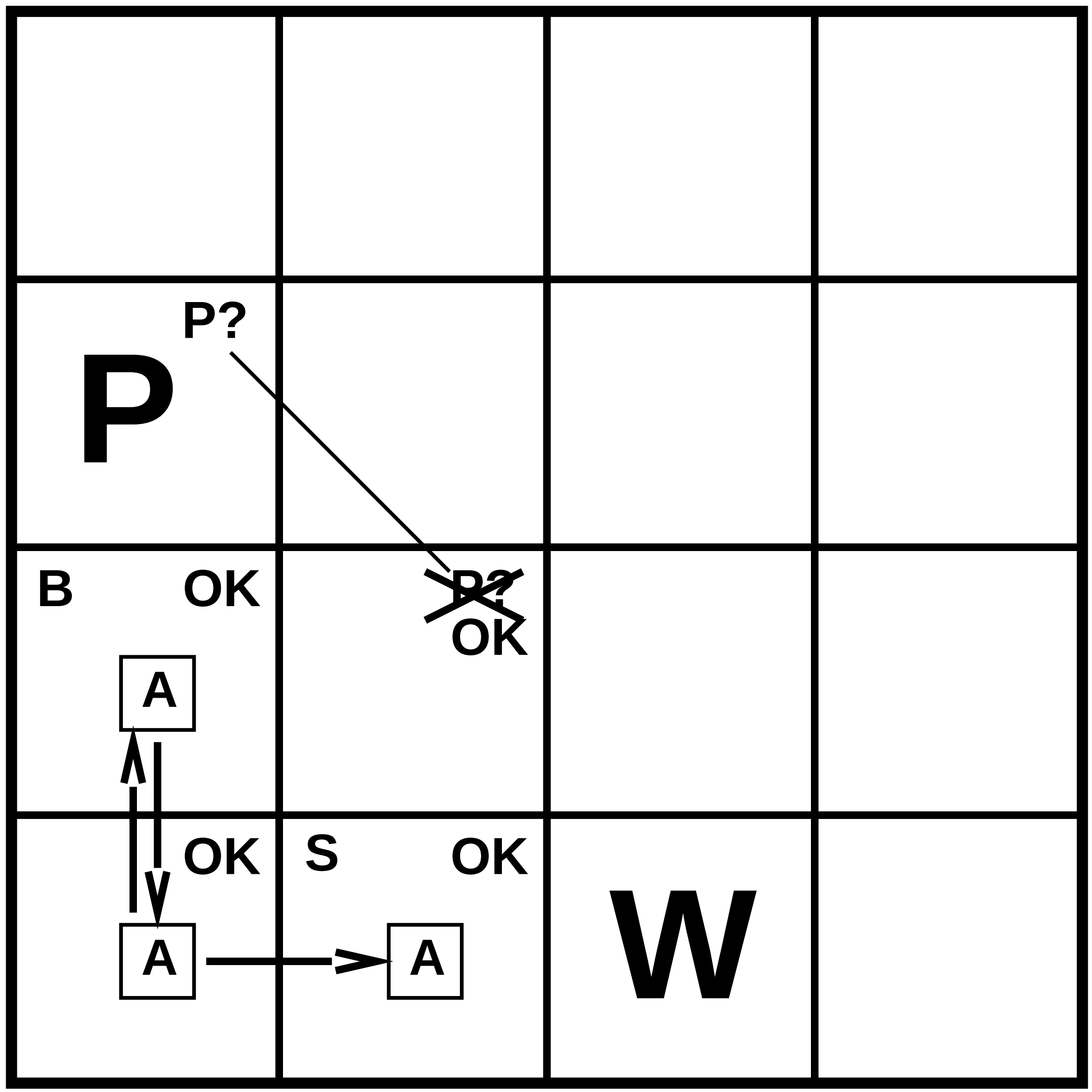

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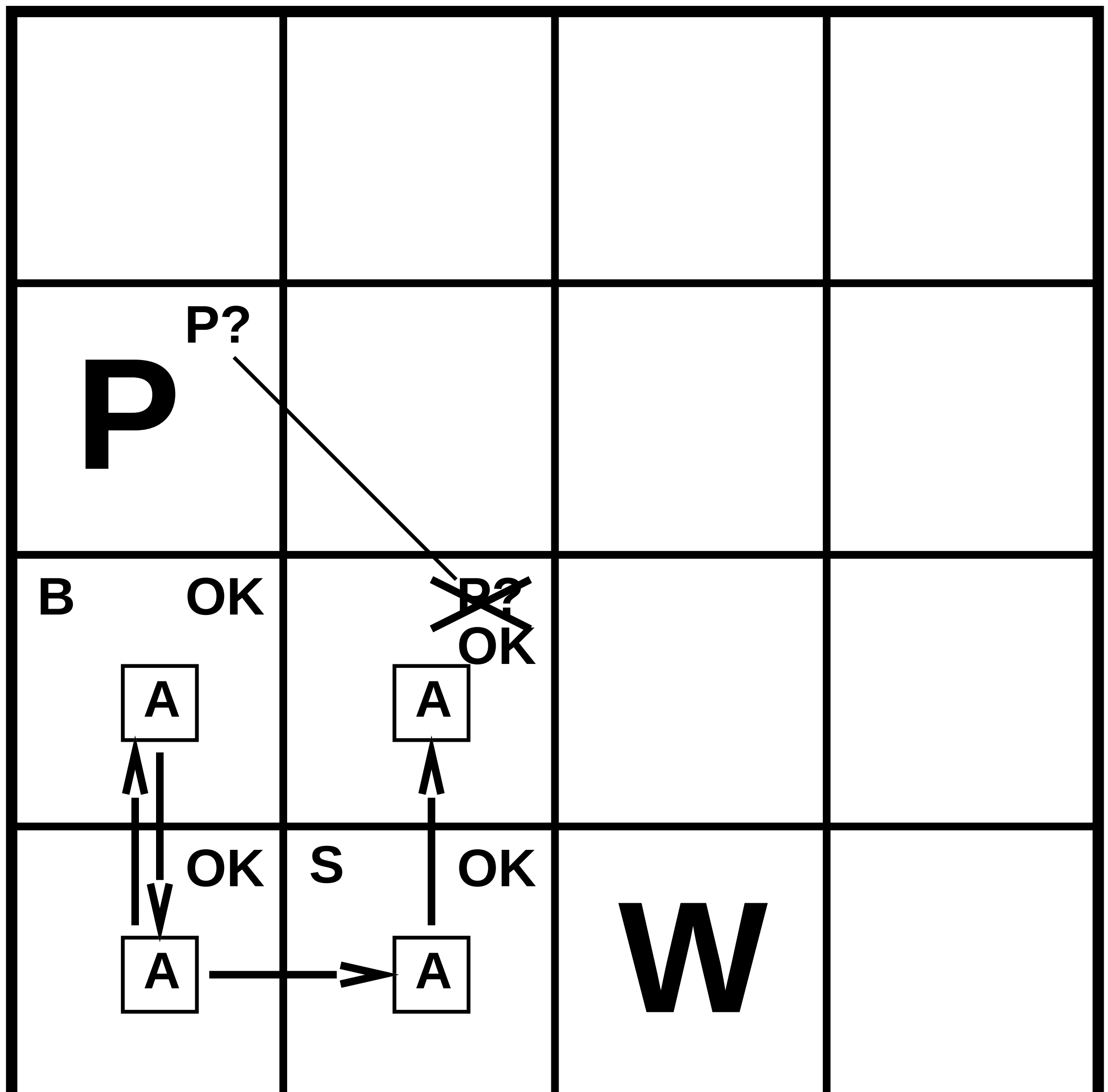

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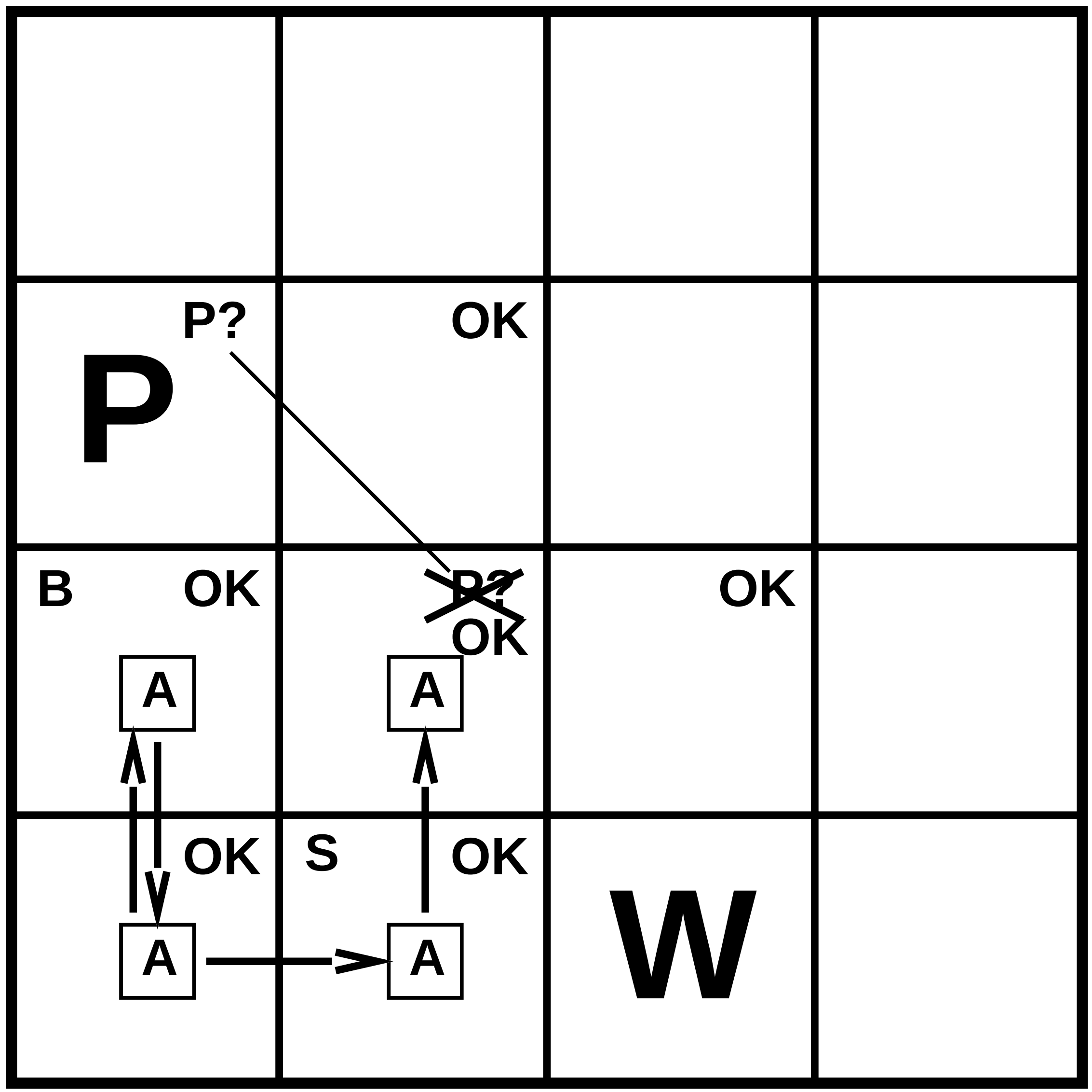

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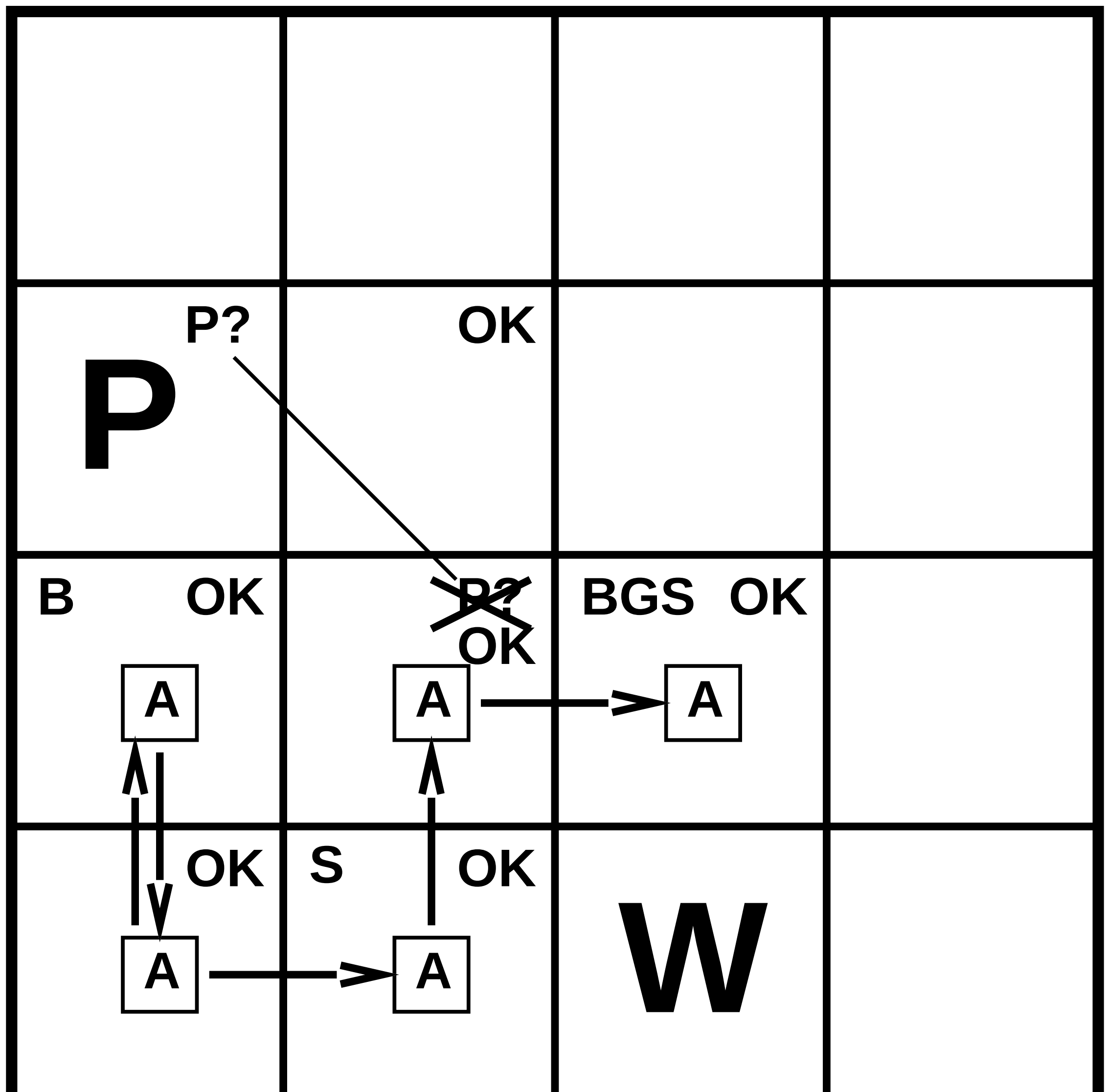
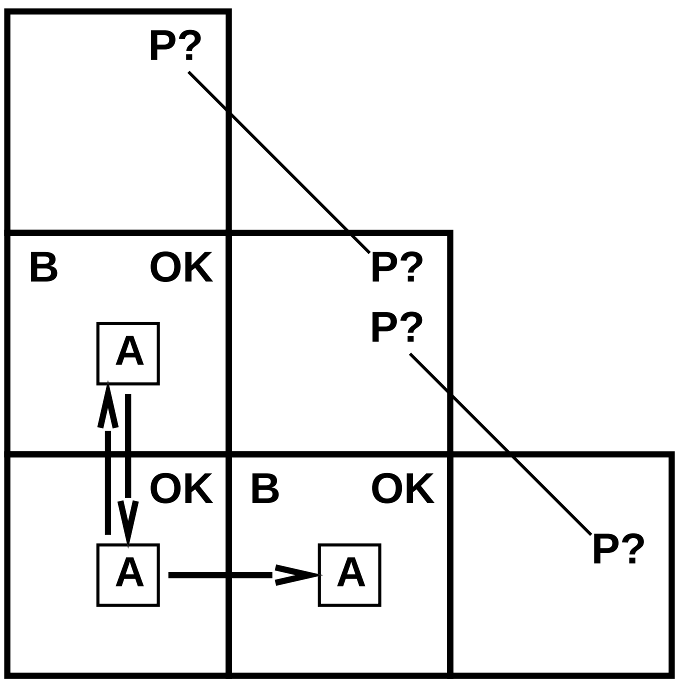


Problematic Situations



Problem

Breeze in (1,2) and (2,1)

$\Rightarrow$ **no safe actions**

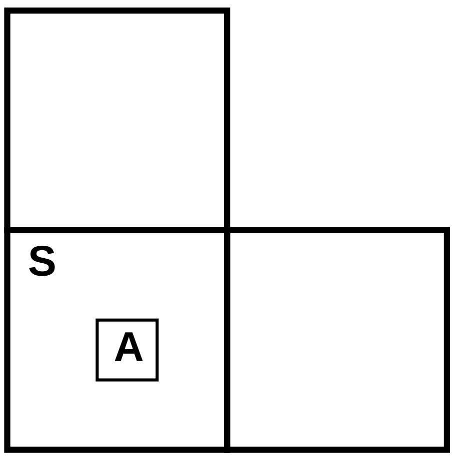
Possible solution

Assuming pits uniformly distributed:

(2,2) has pit with probability 0.86

(1,3) and (3,1) have pit with probab. 0.31

Problematic Situations



Problem

Smell in (1,1)
$\Rightarrow$ no safe actions

Possible solution

Strategy of coercion:

shoot straight ahead
wumpus was there $\Rightarrow$ dead $\Rightarrow$ safe
wumpus wasn't there $\Rightarrow$ safe

Logic in General

Logics

**Formal languages for representing information,
such that conclusions can be drawn**

Syntax

Defines the sentences in the language

Semantics

**Defines the "meaning" of sentences;
i.e., defines truth of a sentence in a world**

Example: Language of Arithmetic

Syntax

$x+2 \geq y$ **is a sentence**

$x2+y>$ **is not a sentence**

Semantics

$x+2 \geq y$ **is true iff the number** $x+2$ **is no less than the number** y

$x+2 \geq y$ **is true in a world where** $x=7,\ y=1$

$x+2 \geq y$ **is false in a world where** $x=0,\ y=6$

Entailment

Definition

Knowledge base KB entails sentence α
if and only if
α is true in all worlds where KB is true

Notation

$$KB \models \alpha$$

Note

Entailment is a relationship between sentences (i.e., syntax) that is based on semantics

Entailment

Example

The KB containing “the shirt is green” and “the shirt is striped” entails “the shirt is green or the shirt is striped”

Example

$x+y=4$ **entails** $4=x+y$

Models

Intuition

Models are formally structured worlds,
with respect to which truth can be evaluated

Definition

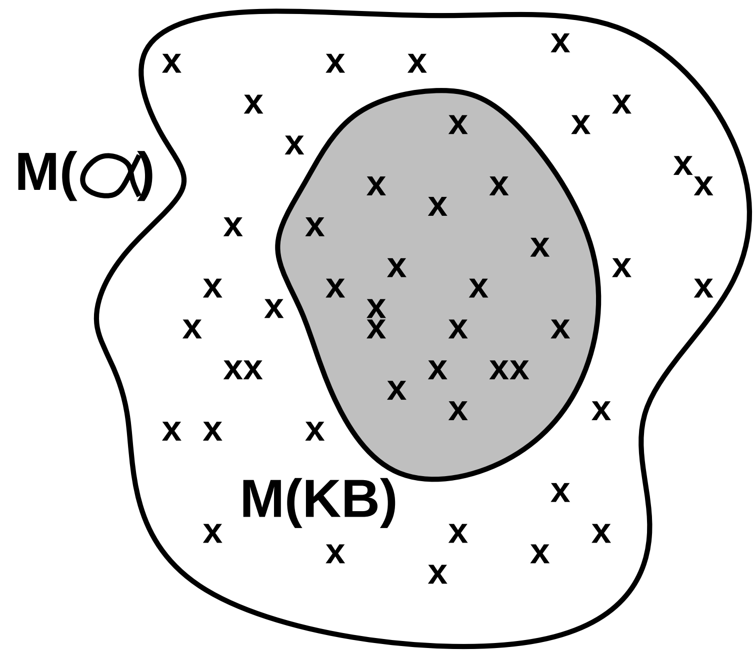
m **is a model of a sentence** α **if** α **is true in** m

$M(\alpha)$ **is the set of all models of** α

Note

$KB \models \alpha$ **if and only if** $M(KB) \subseteq M(\alpha)$

Models: Example



KB = **The shirt is green and striped**

α = **The shirt is green**

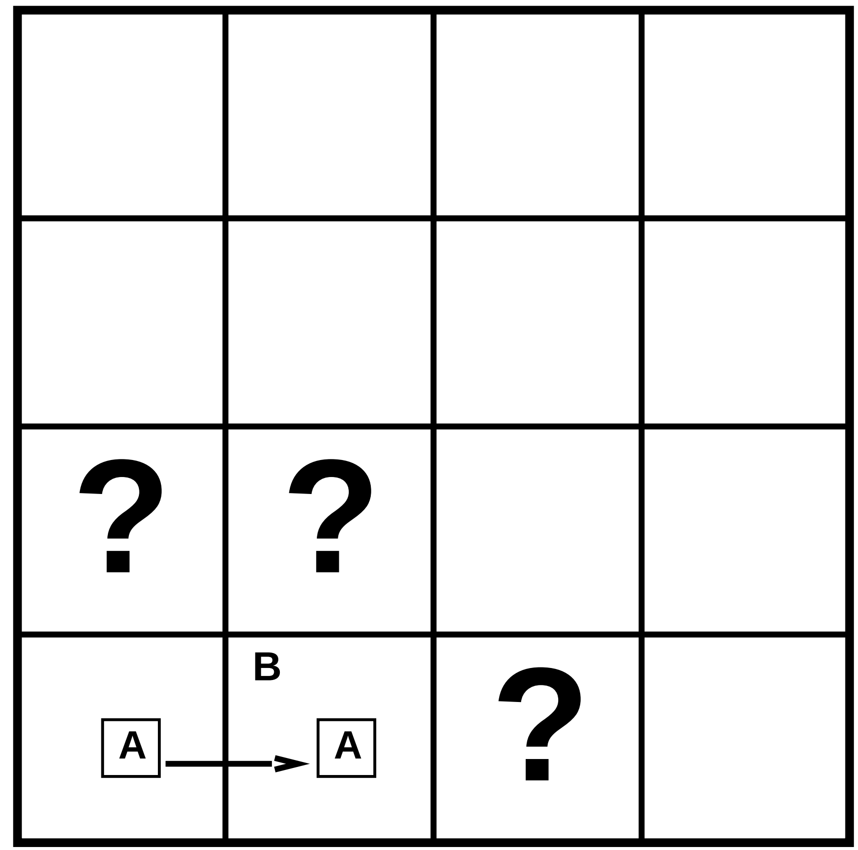
Entailment in the Wumpus World

Situation after

detecting nothing in [1,1],
moving right,
breeze in [2,1]

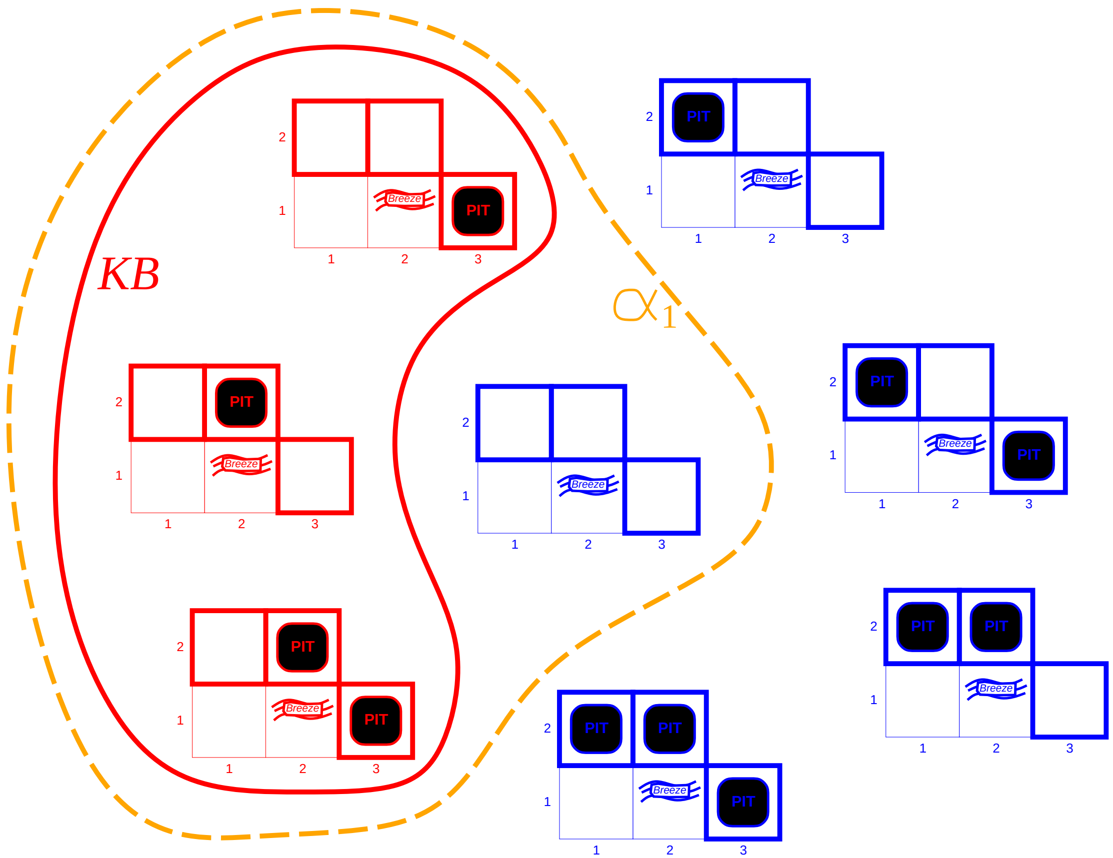
Consider possible models for "?"s (considering only pits)

3 Boolean choices
8 possible models



Wumpus Models

$KB \models \alpha_1$

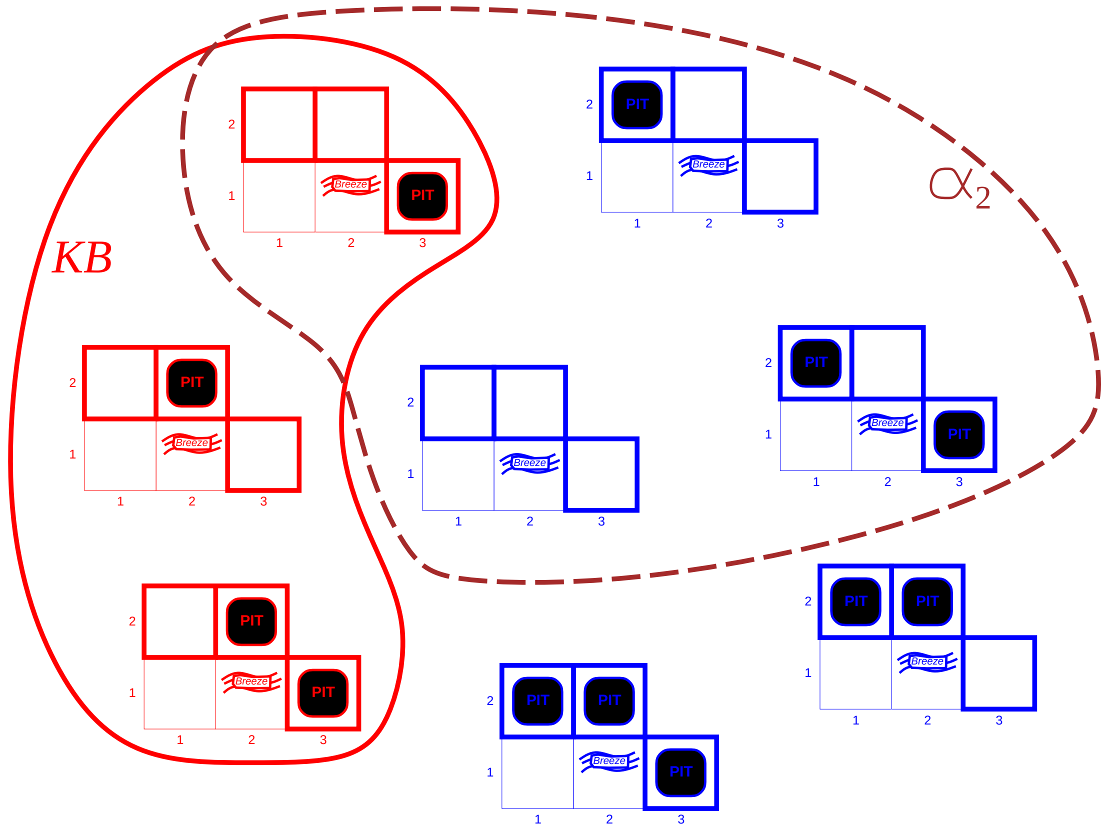


KB = **wumpus-world rules + observations**

α_1 = **"[1,2] is safe"**

Wumpus Models

$KB \not\models \alpha_2$



KB = **wumpus-world rules + observations**

α_2 = **“[2,2] is safe”**

Inference

Definition

$$KB \vdash_i \alpha$$

means

sentence α can be derived from KB by inference procedure i

Soundness (of i)

Whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

Completeness (of i)

Whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

Preview

First-order Logic

We will define a logic (first-order logic) that

- **is expressive enough to say almost anything of interest, and**
- **for which there exists a sound and complete inference procedure.**

That is, the procedure will answer any question whose answer follows from what is known by the KB.

Propositional Logic: Syntax

Definition

- **Propositional symbols**

  $$A,\ B,\ P_1,\ P_2,\ ShirtIsGreen,\quad \textbf{etc.}$$

  are (atomic) sentences

- **If S, S_1, S_2 are sentences, then**

  $$\begin{array}{ll} \neg S & (negation) \\ S_1 \wedge S_2 & (conjunction) \\ S_1 \vee S_2 & (disjunction) \\ S_1 \Rightarrow S_2 & (implication) \\ S_1 \Leftrightarrow S_2 & (equivalence) \end{array}$$

  are sentences

Propositional logic: Semantics

Propositional Models

Each model specifies true/false for each proposition symbol

Example	A	B	C	**(For three symbols, there are 8 possible models)**
	true	**true**	**false**	

Rules for evaluating truth with respect to a model

$\neg S$	**is true iff**	S **is false**		
$S_1 \wedge S_2$	**is true iff**	S_1 **is true**	**and**	S_2 **is true**
$S_1 \vee S_2$	**is true iff**	S_1 **is true**	**or**	S_2 **is true**
$S_1 \Rightarrow S_2$	**is true iff**	S_1 **is false**	**or**	S_2 **is true**
$S_1 \Leftrightarrow S_2$	**is true iff**	S_1 **and** S_2 **have the same truth value**		

Truth Tables for Connectives

A	B	$\neg A$	$A \wedge B$	$A \vee B$	$A \Rightarrow B$	$A \Leftrightarrow B$
false	**false**	**true**	**false**	**false**	**true**	**true**
false	**true**	**true**	**false**	**true**	**true**	**false**
true	**false**	**false**	**false**	**true**	**false**	**false**
true	**true**	**false**	**true**	**true**	**true**	**true**

Wumpus World Sentences

Propositional symbols

$P_{i,j}$ **means: "there is a pit in** $[i, j]$**"**
$B_{i,j}$ **means: "there is a breeze in** $[i, j]$**"**

$$\neg P_{1,1} \qquad \neg B_{1,1} \qquad B_{2,1}$$

Sentences

"Pits cause breezes in adjacent squares"

$$P_{1,2} \Rightarrow (B_{1,1} \wedge B_{1,3} \wedge B_{2,2})$$

"A square is breezy if and only if there is an adjacent pit"

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$
$$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$

Propositional Inference: Enumeration Method

Example

$$\alpha = A \vee B \qquad KB = (A \vee C) \wedge (B \vee \neg C)$$

Checking that $KB \models \alpha$

A	B	C	$A \vee C$	$B \vee \neg C$	KB	α
false	**false**	**false**	**false**	**true**	**false**	**false**
false	**false**	**true**	**true**	**false**	**false**	**false**
false	**true**	**false**	**false**	**true**	**false**	**true**
false	**true**	**true**	**true**	**true**	**true**	**true**
true	**false**	**false**	**true**	**true**	**true**	**true**
true	**false**	**true**	**true**	**false**	**false**	**true**
true	**true**	**false**	**true**	**true**	**true**	**true**
true	**true**	**true**	**true**	**true**	**true**	**true**

Note

Table has 2^n rows for n symbols

Logical Equivalence

Definition

Two sentences are logically equivalent, denoted by

$$\alpha \equiv \beta$$

iff they are true in the same models, i.e., iff:

$$\alpha \models \beta \quad \textbf{and} \quad \beta \models \alpha$$

Example

$$(A \Rightarrow B) \quad \equiv \quad (\neg B \Rightarrow \neg A) \qquad \textbf{(contraposition)}$$

Logical Equivalence

Theorem

If

- $\alpha \equiv \beta$
- γ **is the result of replacing a subformula** α **of** δ **by** β**,**

then $\gamma \equiv \delta$

Example

$$A \vee B \quad \equiv \quad B \vee A$$

implies

$$(C \wedge (A \vee B)) \Rightarrow D \quad \equiv \quad (C \wedge (B \vee A)) \Rightarrow D$$

Important Equivalences

$(\alpha \wedge \beta)$	$\equiv$	$(\beta \wedge \alpha)$	**commutativity of $\wedge$**
$(\alpha \vee \beta)$	$\equiv$	$(\beta \vee \alpha)$	**commutativity of $\vee$**
$((\alpha \wedge \beta) \wedge \gamma)$	$\equiv$	$(\alpha \wedge (\beta \wedge \gamma))$	**associativity of $\wedge$**
$((\alpha \vee \beta) \vee \gamma)$	$\equiv$	$(\alpha \vee (\beta \vee \gamma))$	**associativity of $\vee$**
$(\alpha \wedge \alpha)$	$\equiv$	α	**idempotence for $\wedge$**
$(\alpha \vee \alpha)$	$\equiv$	α	**idempotence for $\vee$**
$\neg\neg\alpha$	$\equiv$	α	**double-negation elimination**
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\beta \Rightarrow \neg\alpha)$	**contraposition**
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\alpha \vee \beta)$	**implication elimination**
$(\alpha \Leftrightarrow \beta)$	$\equiv$	$((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$	**equivalence elimination**
$\neg(\alpha \wedge \beta)$	$\equiv$	$(\neg\alpha \vee \neg\beta)$	**de Morgan's rules**
$\neg(\alpha \vee \beta)$	$\equiv$	$(\neg\alpha \wedge \neg\beta)$	**de Morgan's rules**
$(\alpha \wedge (\beta \vee \gamma))$	$\equiv$	$((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	**distributivity of $\wedge$ over $\vee$**
$(\alpha \vee (\beta \wedge \gamma))$	$\equiv$	$((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	**distributivity of $\vee$ over $\wedge$**

The Logical Constants *true* and *false*

Semantics

true **evaluates to** *true* **in all models**

false **evaluates to** *false* **in all models**

Important equivalences with *true* and *false*

$$(\alpha \wedge \neg\alpha) \equiv \mathit{false}$$

$$(\alpha \vee \neg\alpha) \equiv \mathit{true} \qquad \textbf{tertium non datur}$$

$$(\alpha \wedge \mathit{true}) \equiv \alpha$$

$$(\alpha \wedge \mathit{false}) \equiv \mathit{false}$$

$$(\alpha \vee \mathit{true}) \equiv \mathit{true}$$

$$(\alpha \vee \mathit{false}) \equiv \alpha$$

Validity

Definition

A sentence is valid if it is true in all models

Examples

$$A \vee \neg A, \qquad A \Rightarrow A, \qquad (A \wedge (A \Rightarrow B)) \Rightarrow B$$

Deduction Theorem **(connects inference and validity)**

$$KB \models \alpha \quad \textbf{if and only if} \quad KB \Rightarrow \alpha \quad \textbf{is valid}$$

Satisfiability

Definition

A sentence is satisfiable if it is true in some model

Examples

$$A \vee B, \qquad A, \qquad A \wedge (A \Rightarrow B)$$

Definition

A sentence is unsatisfiable if it is true in no models,
i.e., if it is not satisfiable

Example

$$A \wedge \neg A$$

Satisfiability

Theorem (connects validity and unsatisfiability)

α is valid if and only if $\neg\alpha$ is unsatisfiable

Theorem (connects inference and unsatisfiability)

$KB \models \alpha$ if and only if $(KB \wedge \neg\alpha)$ is unsatisfiable

Note

Validity and inference can be proved by **reductio ad absurdum**

Two Kinds of Proof Methods

1. Application of inference rules

Legitimate (sound) generation of new sentences from old

Construction of / search for a proof
(proof = sequence of inference rule applications)

Properties

Typically requires translation of sentences into a normal form

Different kinds

Tableau calculus, resolution, forward/backward chaining, ...

Two Kinds of Proof Methods

2. Model checking

Construction of / search for a satisfying model

Different kinds

- **Truth table enumeration (always exponential number of symbols)**
- **Improved backtracking search for models**
  e.g.: Davis-Putnam-Logemann-Loveland
- **Heuristic search in model space (sound but incomplete)**
  e.g.: hill-climbing algorithms

Normal Forms

Literal

A literal is

- **an atomic sentence (propositional symbol), or**
- **the negation of an atomic sentence**

Clause

A disjunction of literals

Conjunctive Normal Form (CNF)

A conjunction of disjunctions of literals,
i.e., a conjunction of clauses

Example

$$(A \vee \neg B) \;\wedge\; (B \vee \neg C \vee \neg D)$$

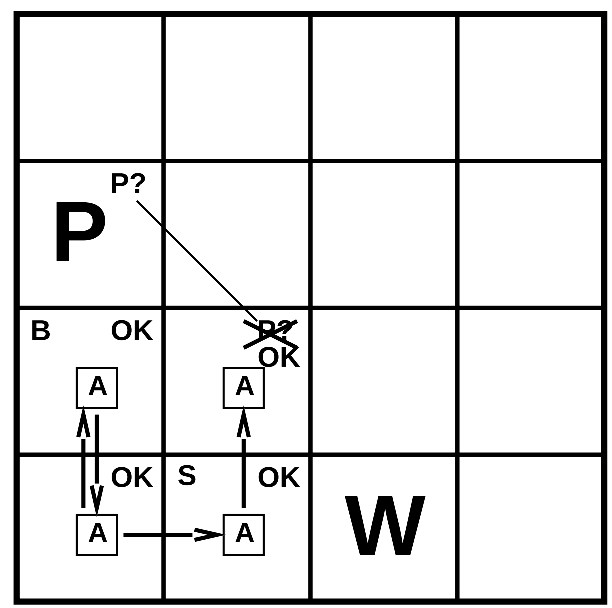
Resolution

Inference rule

$$\frac{P_1 \vee \cdots \vee P_{i-1} \vee Q \vee P_{i+1} \vee \ldots \vee P_k \qquad R_1 \vee \cdots \vee R_{j-1} \vee \neg Q \vee R_{j+1} \vee \ldots \vee R_n}{P_1 \vee \cdots \vee P_{i-1} \vee P_{i+1} \vee \ldots \vee P_k \ \vee \ R_1 \vee \cdots \vee R_{j-1} \vee R_{j+1} \vee \ldots \vee R_n}$$

Example

$$\frac{P_{1,3} \vee P_{2,2} \qquad \neg P_{2,2}}{P_{1,3}}$$



Resolution

Correctness theorem

Resolution is sound and complete for propositional logic,

i.e., given a formula α in CNF (conjunction of clauses):

> α **is unsatisfiable**
> **iff**
> **the empty clause can be derived from α with resolution**

Conversion to CNF

0. Given

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \equiv \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$

$$(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \;\wedge\; ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \;\wedge\; (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$$

3. Move $\neg$ inwards using de Morgan's rules (and double-negation)

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \;\wedge\; ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$$

4. Apply distributivity law ($\vee$ over $\wedge$) and flatten

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \;\wedge\; (\neg P_{1,2} \vee B_{1,1}) \;\wedge\; (\neg P_{2,1} \vee B_{1,1})$$

Resolution Example

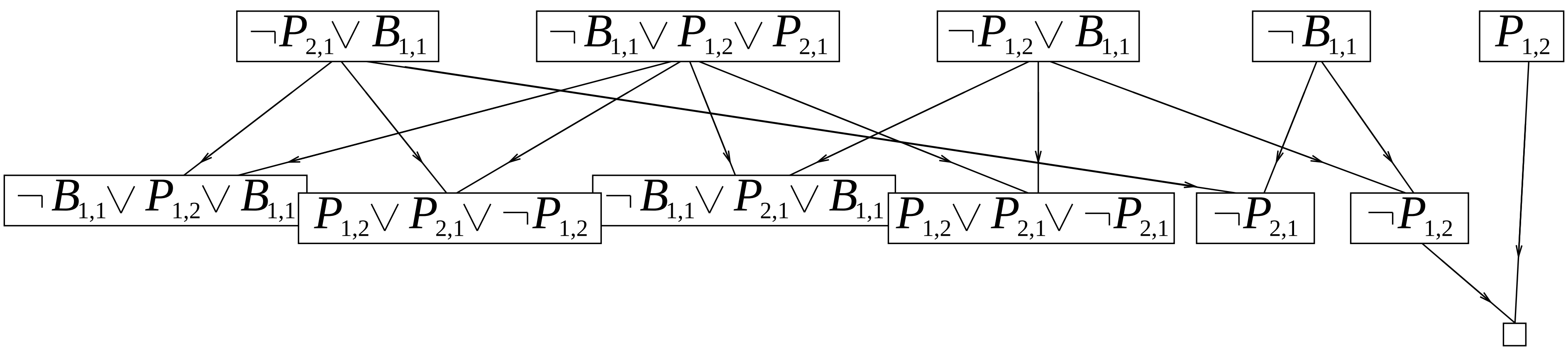
Given

$$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$$

$$\alpha = \neg P_{1,2}$$

Resolution proof for $KB \models \alpha$

Derive empty clause □ from $KB \wedge \neg\alpha$ in CNF



Summary

- Logical agents apply inference to a knowledge base to derive new information and make decisions
- Basic concepts of logic
  - **syntax**: formal structure of sentences
  - **semantics**: truth of sentences w.r.t. models
  - **entailment**: necessary truth of one sentence given another
  - **inference**: deriving sentences from other sentences
  - **soundess**: derivations produce only entailed sentences
  - **completeness**: derivations can produce all entailed sentences
- Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.
- Resolution is sound and complete for propositional logic
- Propositional logic lacks expressive power