

Timed Automata – Decidability Results



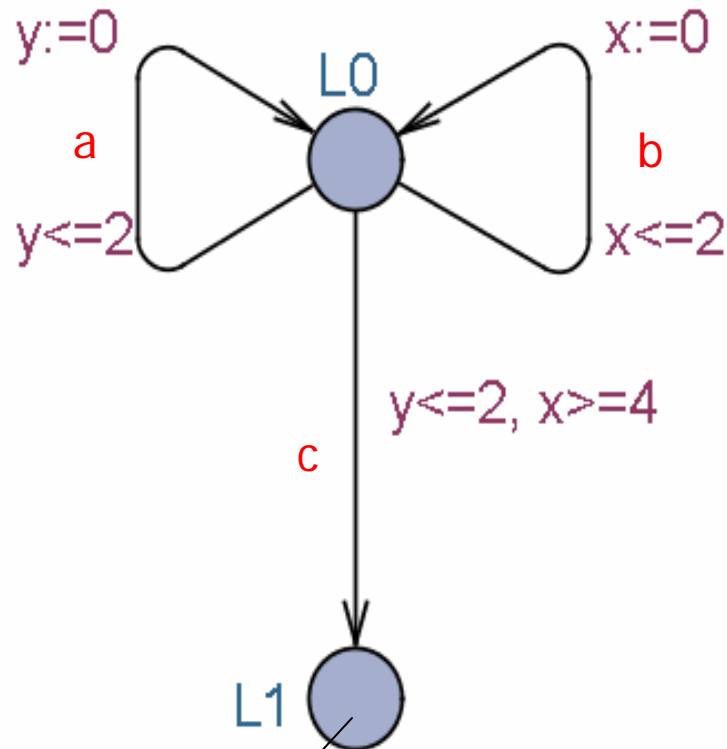
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Decidability ?



Reachable?

OBSTACLE:
 Uncountably infinite
 state space

Derived Relations and Reachability

$$(l, u) \xrightarrow{\delta} (l', u') \quad \text{iff} \quad \exists d > 0. (l, u) \xrightarrow{\epsilon(d)} (l', u').$$

$$(l, u) \xrightarrow{\alpha} (l', u') \quad \text{iff} \quad \exists a \in \text{Act}. (l, u) \xrightarrow{a} (l', u')$$

$$(l, u) \rightsquigarrow (l', u') \quad \text{iff} \quad (l, u) (\xrightarrow{\delta} \cup \xrightarrow{\alpha})^* (l', u')$$

Definition

The set of reachable locations, $\text{Reach}(A)$, of a timed automaton A is defined as:

$$l \in \text{Reach}(A) \equiv^{\Delta} \exists u. (l_0, u_0) \rightsquigarrow (l, u)$$

Time Abstracted Bisimulation

Definition

Let $G \subseteq L$ be a set of goal locations. An equivalence relation R on $L \times \mathbb{R}^C$ is a TAB wrt G if whenever $(l, u)R(n, v)$ the following holds:

1. $l \in G$ iff $n \in G$,
2. whenever $(l, u) \xrightarrow{\delta} (l', u')$ then $(n, v) \xrightarrow{\delta} (n', v')$ with $(l', u')R(n', v')$
3. whenever $(l, u) \xrightarrow{a} (l', u')$ then $(n, v) \xrightarrow{a} (n', v')$ with $(l', u')R(n', v')$

Stable Quotient

Definition

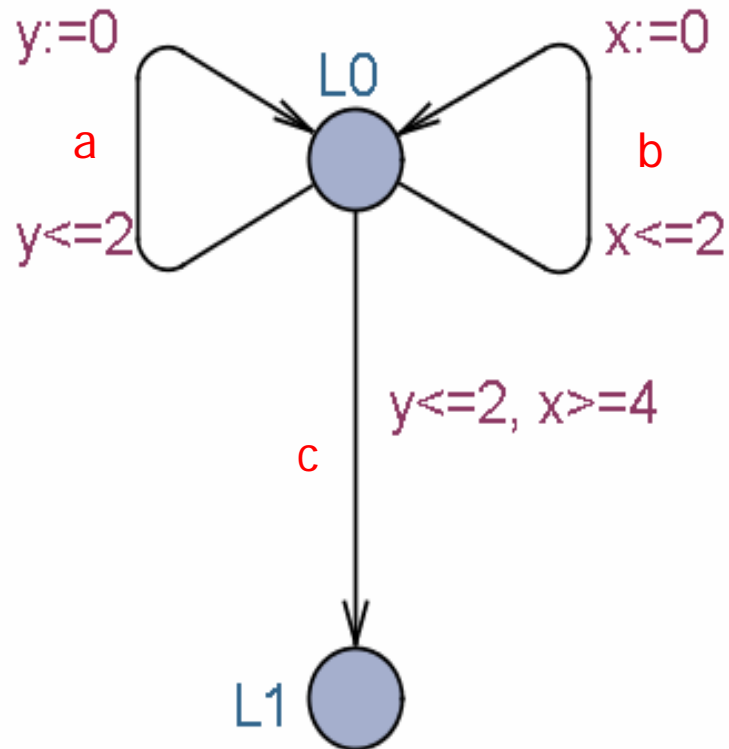
Let R be a TAB wrt G . The induced *quotient* has classes of R , $\pi \in (L \times \mathbb{R}^C / R)$, as states. For classes π, π' the transitions are

- $\pi \xrightarrow{\delta} \pi'$ iff $(l, u) \xrightarrow{\delta} (l', u')$ for some $(l, u) \in \pi, (l', u') \in \pi'$.
- $\pi \xrightarrow{a} \pi'$ iff $(l, u) \xrightarrow{a} (l', u')$ for some $(l, u) \in \pi, (l', u') \in \pi'$.

Theorem

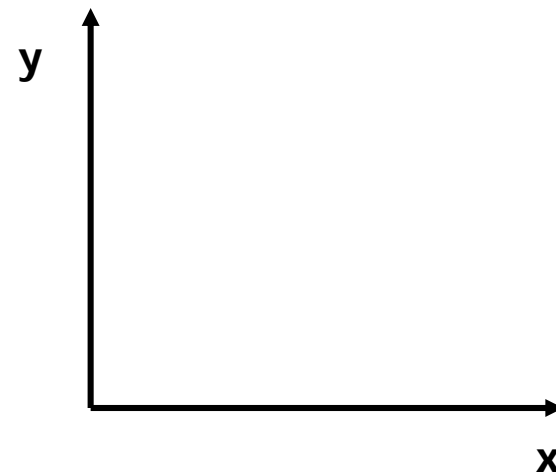
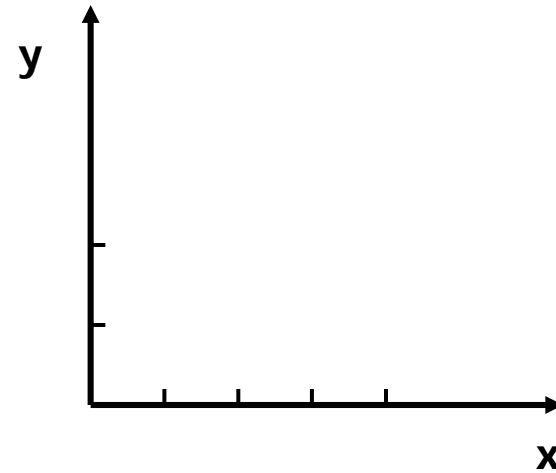
Let R be TAB wrt G . Then, a location from G is reachable iff there exists an equivalence class π of R such that π is reachable in the quotient and π contains a state whose location is in G .

Stable Quotient

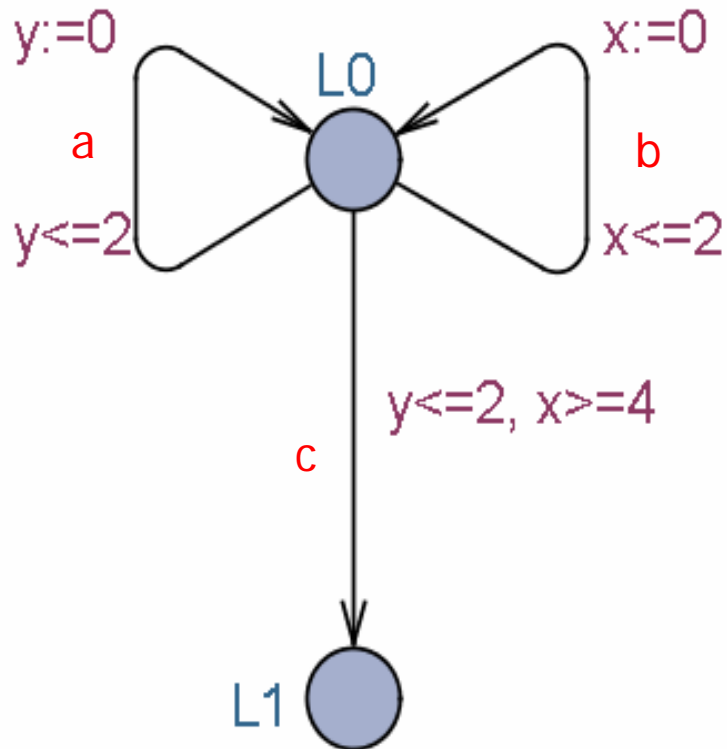


Reachable?

Partitioning

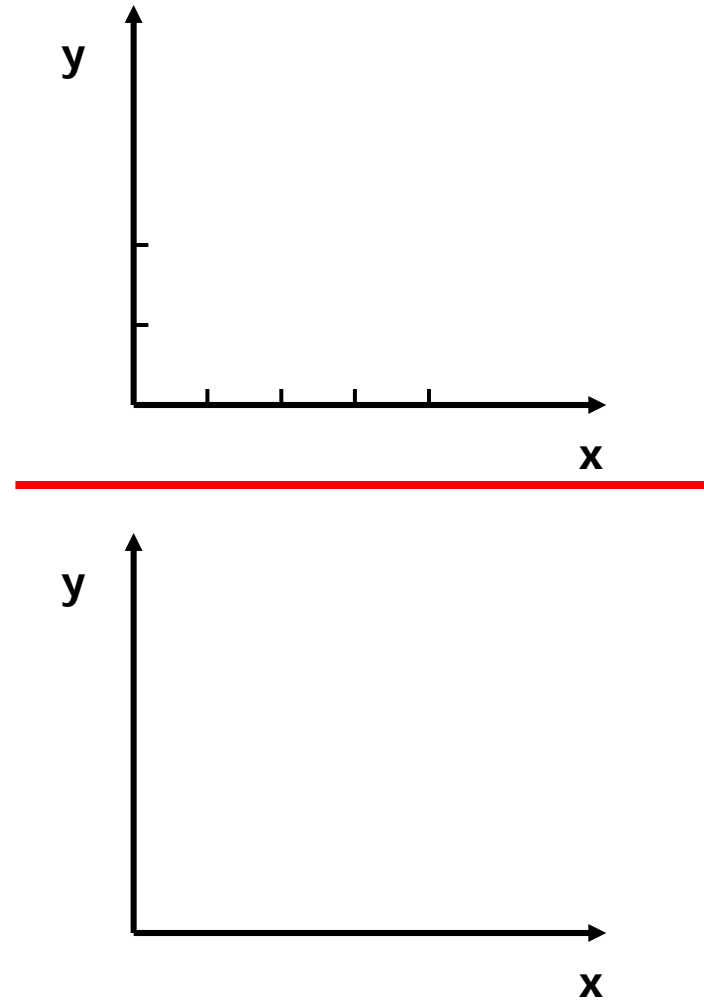


Stable Quotient

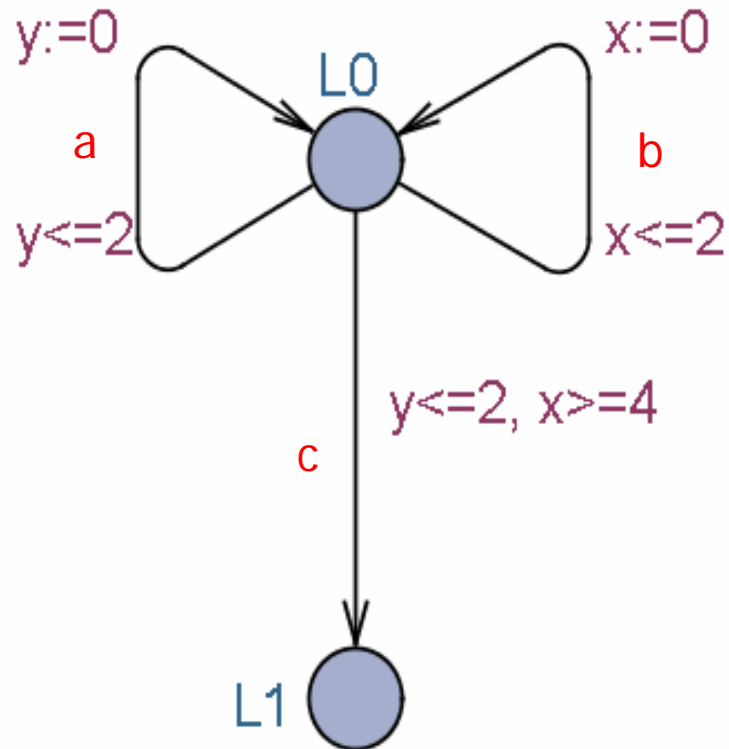


Reachable?

Partitioning

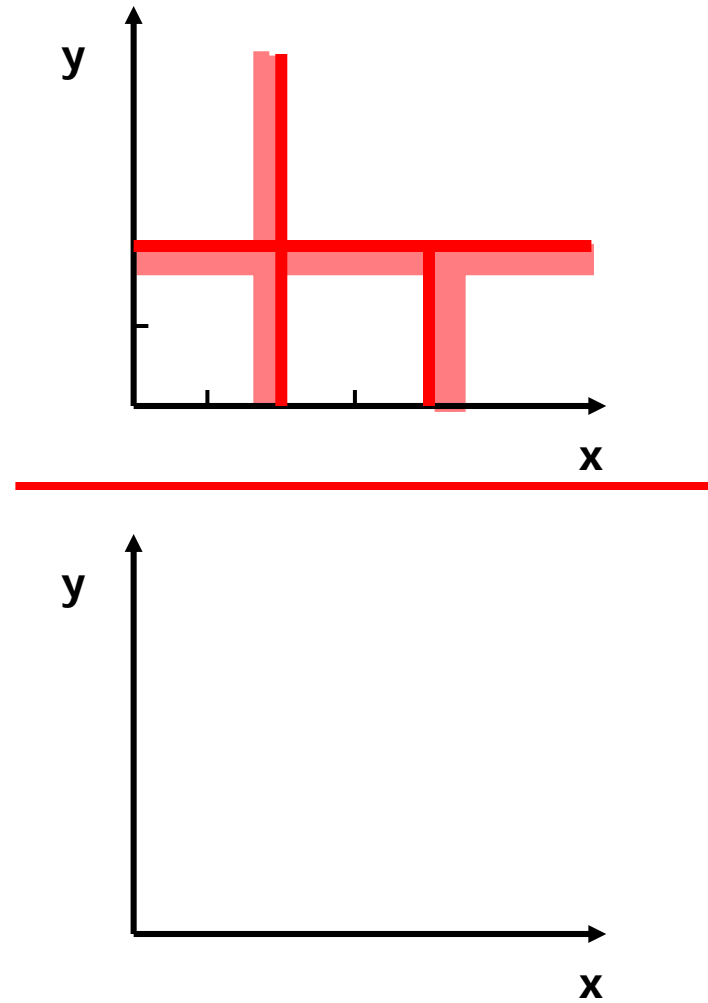


Stable Quotient

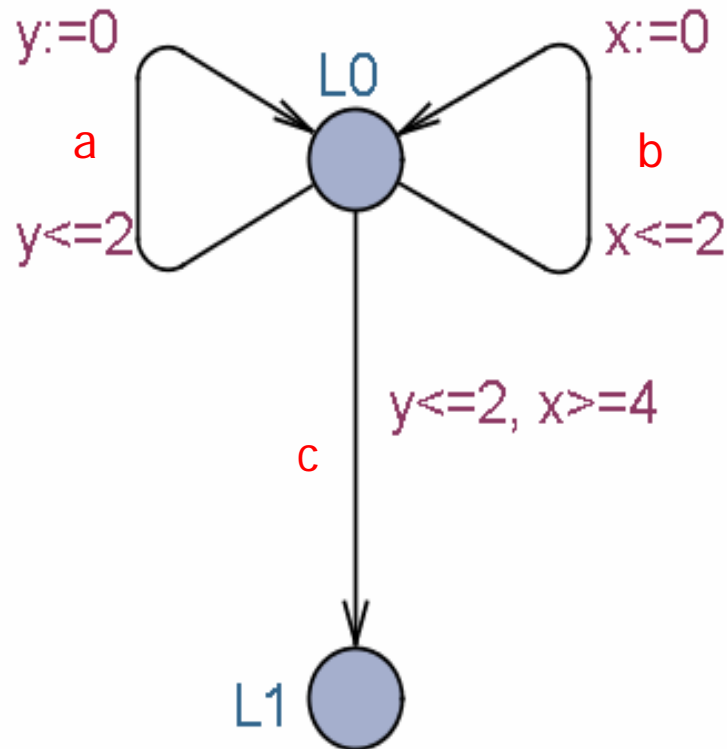


Reachable?

Partitioning

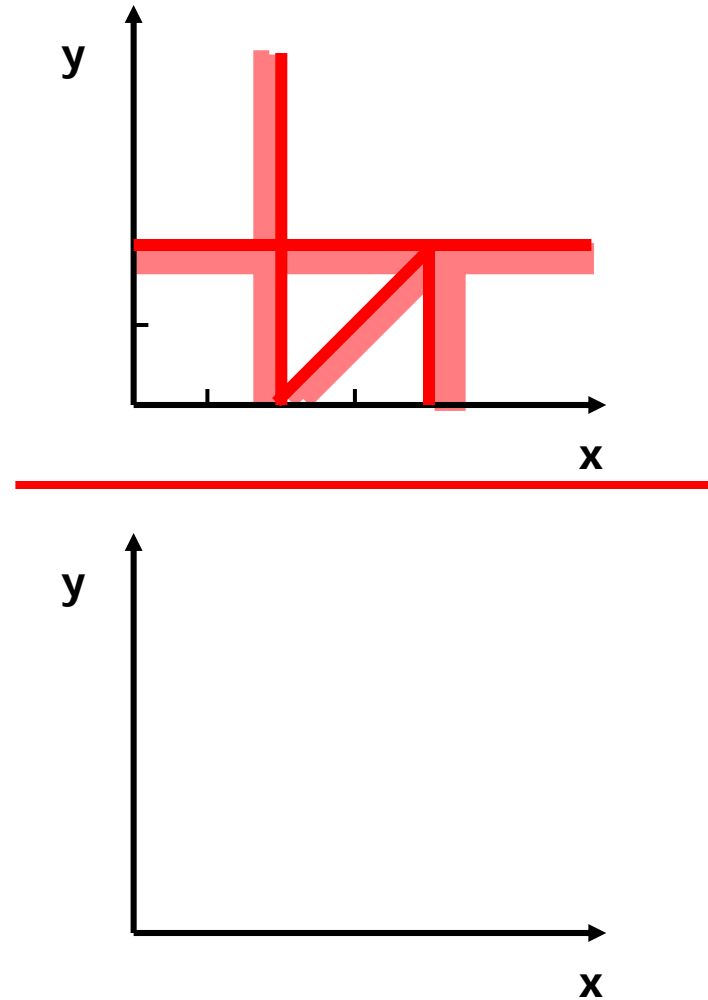


Stable Quotient

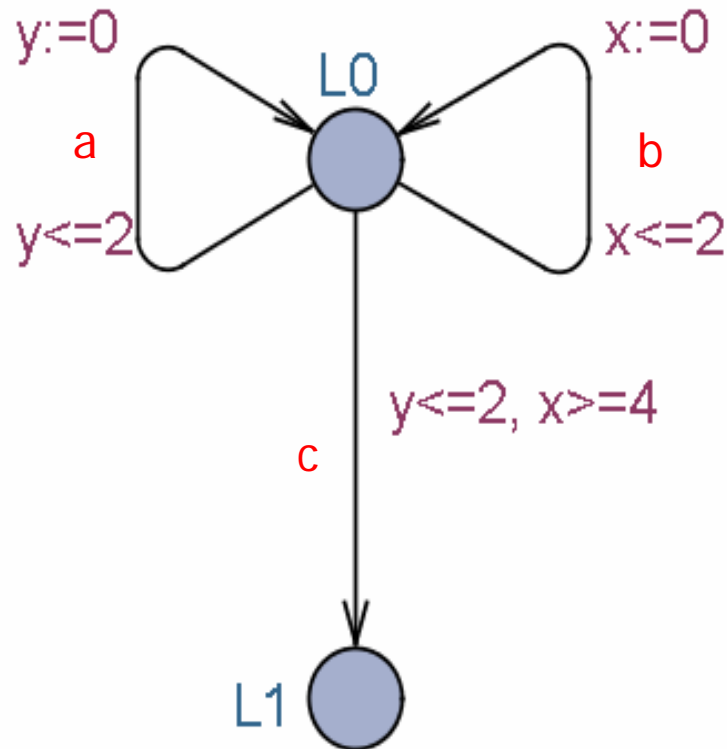


Reachable?

Partitioning

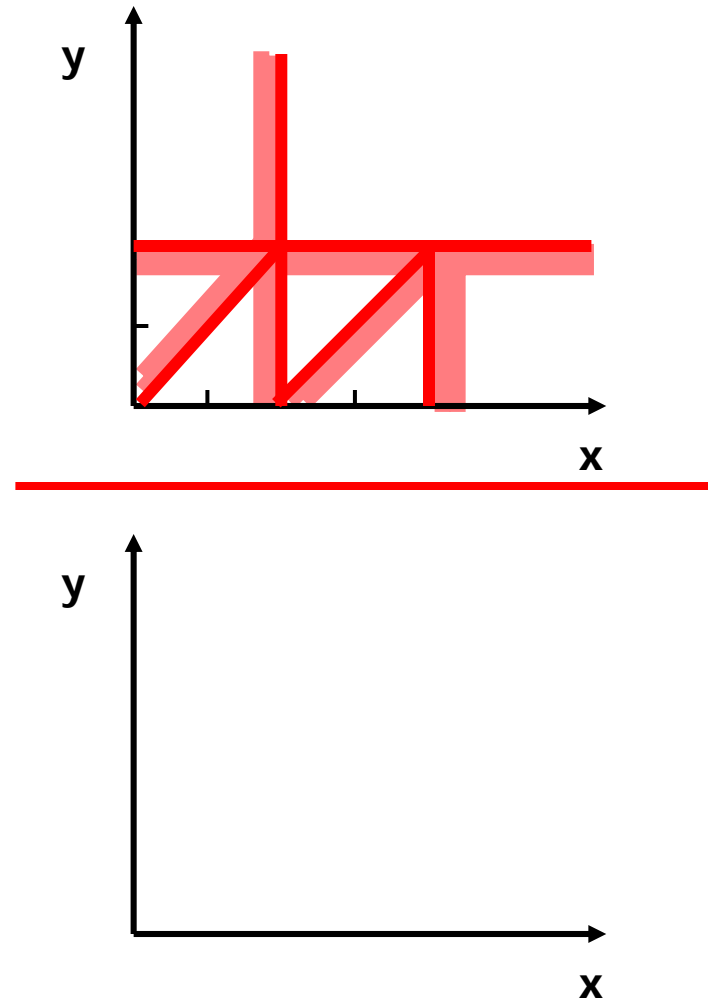


Stable Quotient



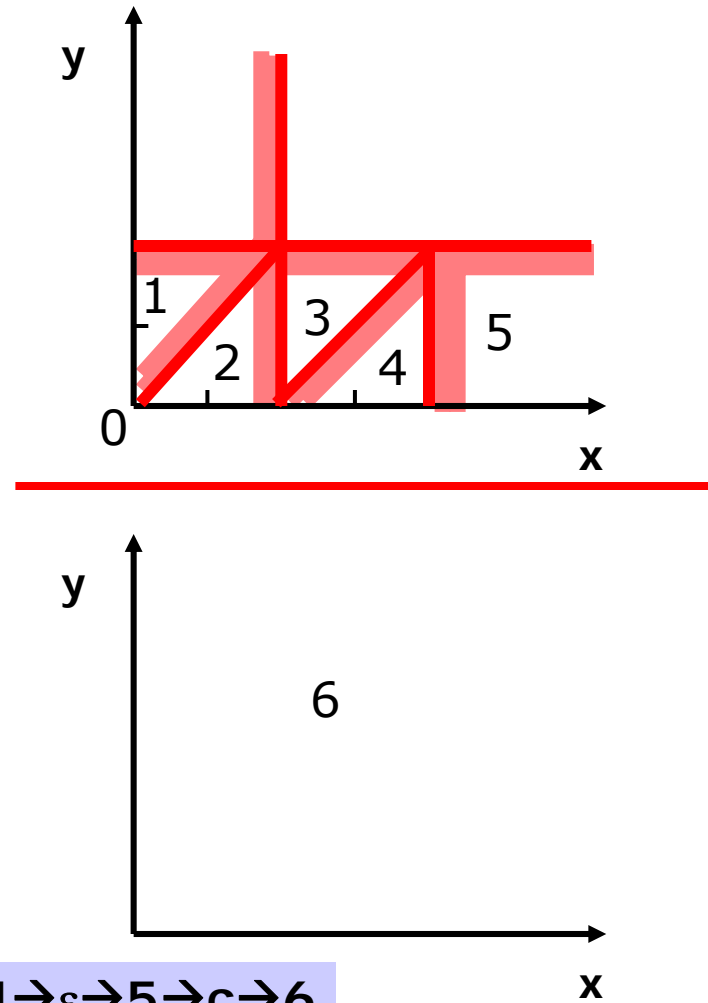
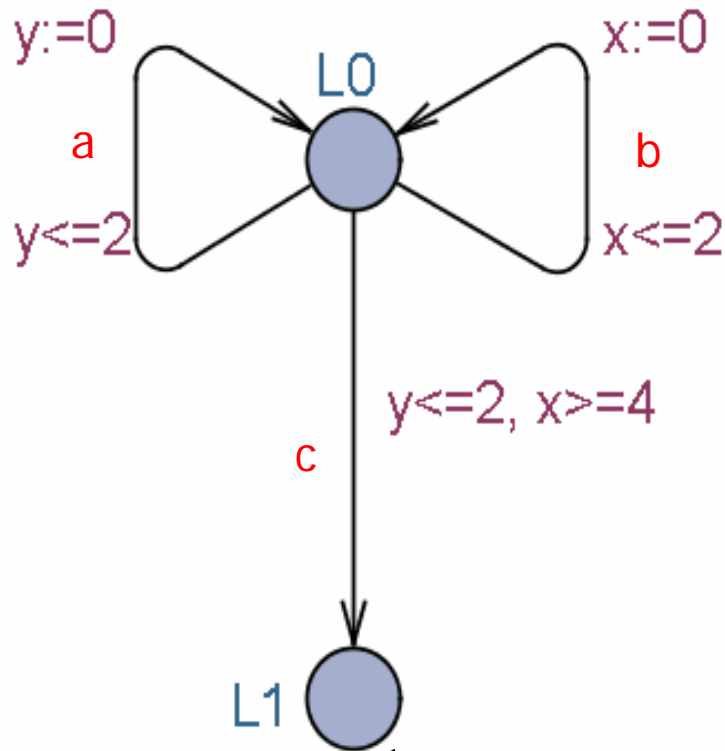
Reachable?

Partitioning



Stable Quotient

Partitioning



Reachable?

$0 \rightarrow \epsilon \rightarrow 1 \rightarrow a \rightarrow 2 \rightarrow \epsilon \rightarrow 3 \rightarrow a \rightarrow 4 \rightarrow \epsilon \rightarrow 5 \rightarrow c \rightarrow 6$

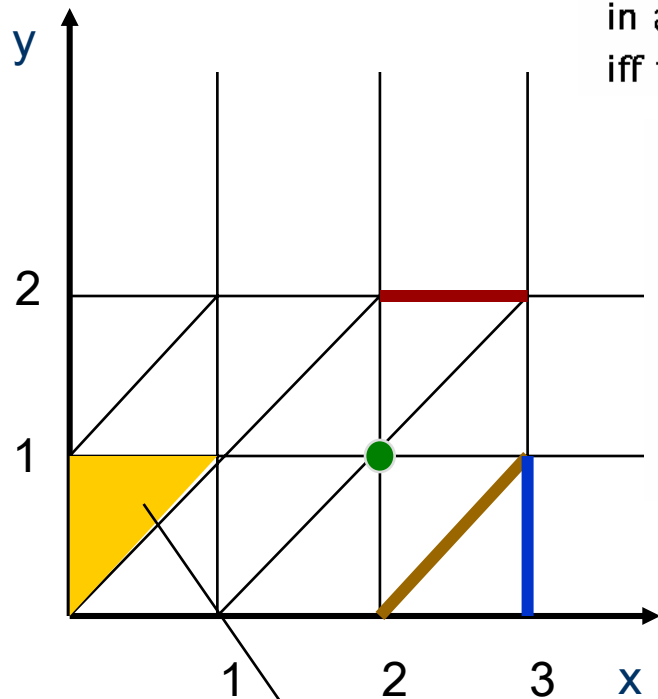
Region Equivalence

For each clock x let c_x be the largest integer with which x is compared in any guard or invariant of A . u and u' are *region equivalent*, $u \cong u'$ iff the following holds:

1. For all $x \in C$, either $\lfloor u(x) \rfloor = \lfloor u'(x) \rfloor$ or $u(x), u'(x) > c_x$;
2. For all $x, y \in C$ with $u(x) \leq c_x$ and $u(y) \leq c_y$,
 $fr(u(x)) \leq fr(u(y))$ iff $fr(u'(x)) \leq fr(u'(y))$;
3. For all $x \in C$ with $u(x) \leq c_x$,
 $fr(u(x)) = 0$ iff $fr(u'(x)) = 0$.

Regions

Finite Partitioning of State Space



For each clock x let c_x be the largest integer with which x is compared in any guard or invariant of A . u and u' are region equivalent, $u \cong u'$ iff the following holds:

1. For all $x \in C$, either $\lfloor u(x) \rfloor = \lfloor u'(x) \rfloor$ or $u(x), u'(x) > c_x$;
2. For all $x, y \in C$ with $u(x) \leq c_x$ and $u(y) \leq c_y$, $fr(u(x)) \leq fr(u(y))$ iff $fr(u'(x)) \leq fr(u'(y))$;
3. For all $x \in C$ with $u(x) \leq c_x$, $fr(u(x)) = 0$ iff $fr(u'(x)) = 0$.

An equivalence class (i.e. a *region*)
in fact there is only a *finite* number of regions!!

Logical Characterization of Regions

Each region may be represented by specifying

1. for every clock x a constraint from

$$\{x = c \mid c = 0, 1, \dots, c_x\} \cup \{c - 1 < x < c \mid c = 1, \dots, c_x\} \cup \{x > c_x\}$$

2. for every pair of clocks x, y such that $c - 1 < x < c$ and $d - 1 < y < d$ appears in 1., whether $fr(x)$ is $<$, $=$ or $>$ than $fr(y)$.

Theorem

The number of regions is $n! \cdot 2^n \cdot \prod_{x \in C} (2c_x + 2)$.

Stability of Regions

Lemma

1. If $u \cong u'$ then $\forall d. \exists d'. u + d \cong u' + d'$;
2. If $u \cong u'$ then* $\forall g \in \mathcal{B}(X). u \models g \Leftrightarrow u' \models g$;
3. If $u \cong u'$ then $\forall r \subseteq C. u[r] \cong u'[r]$.

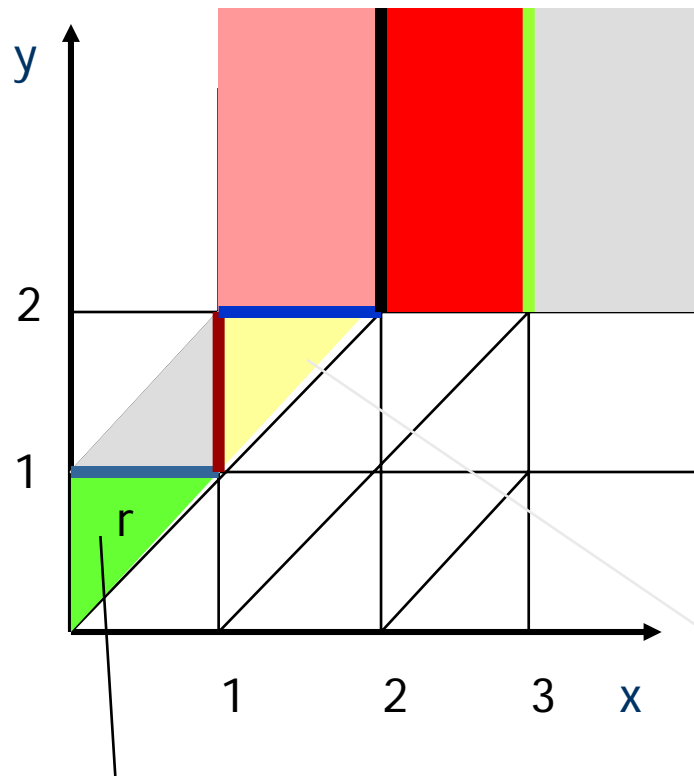
Theorem

Let $(l, u) \cong (l', u')$ iff $l = l'$ and $u \cong u'$. Then \cong is a TAB with respect to any set of goal locations G .

*Here \cong and g should agree on the maximal constants.

Regions

Successor Operation (wrt delay)

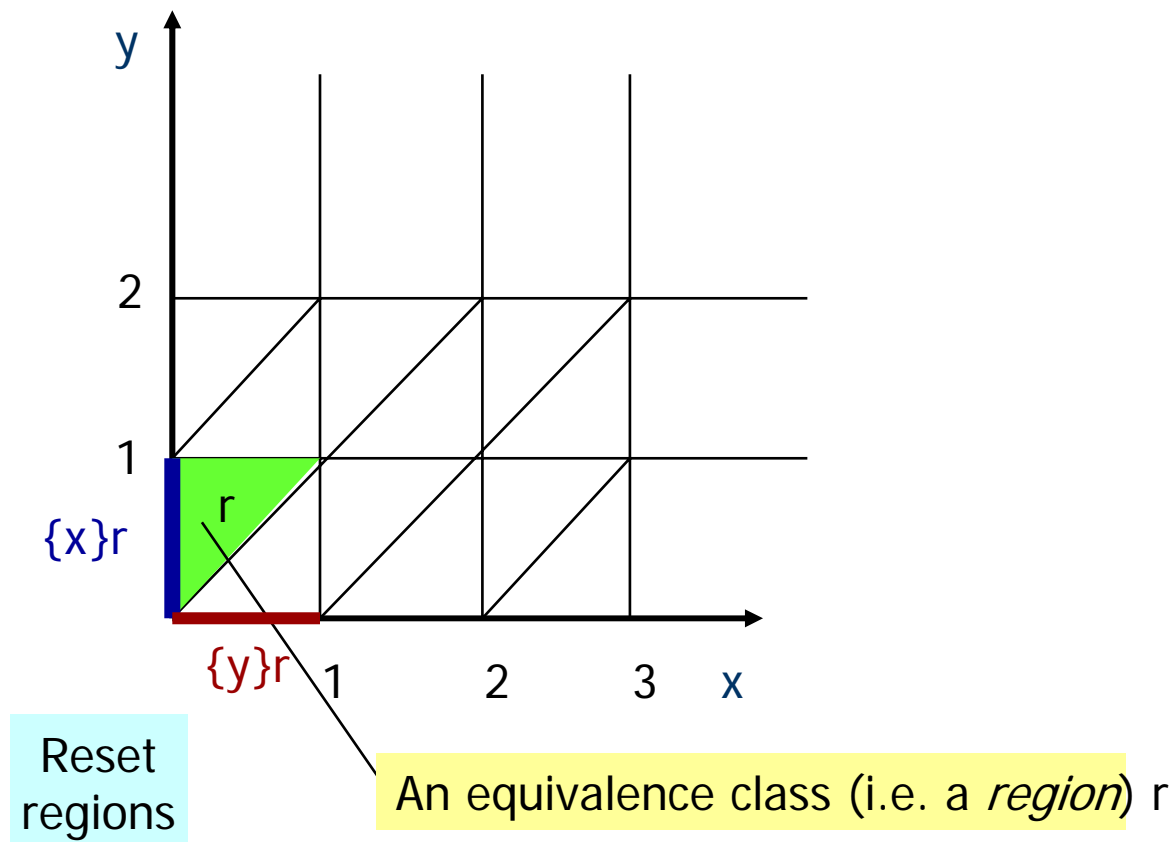


Successor regions, $\text{Succ}(r)$

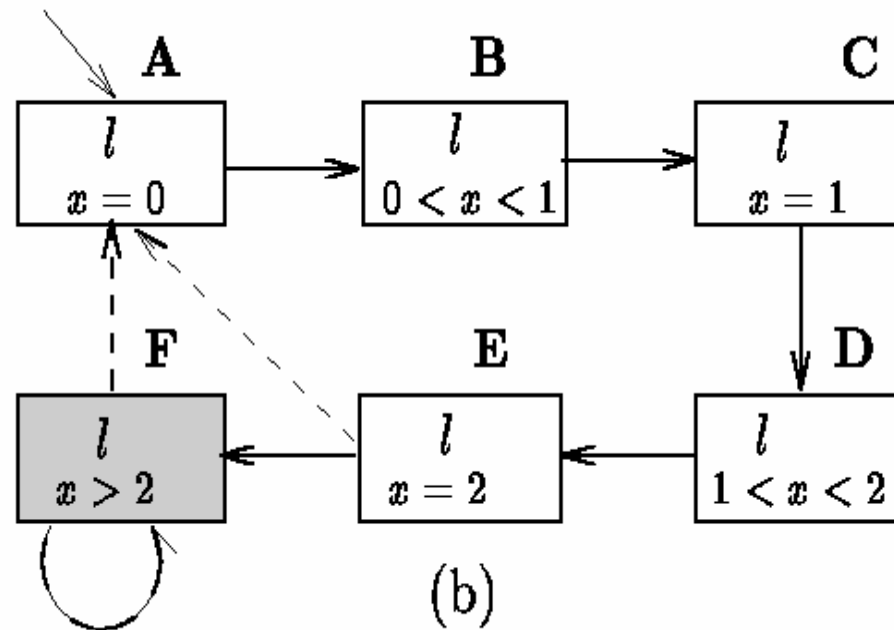
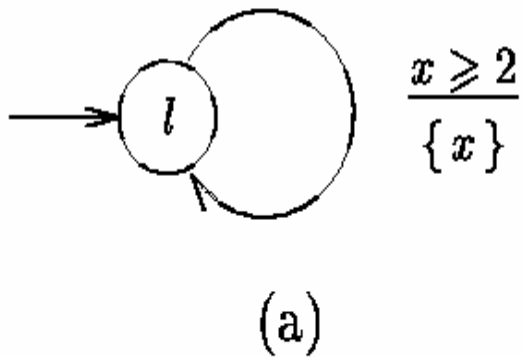
An equivalence class (i.e. a *region*)

Regions

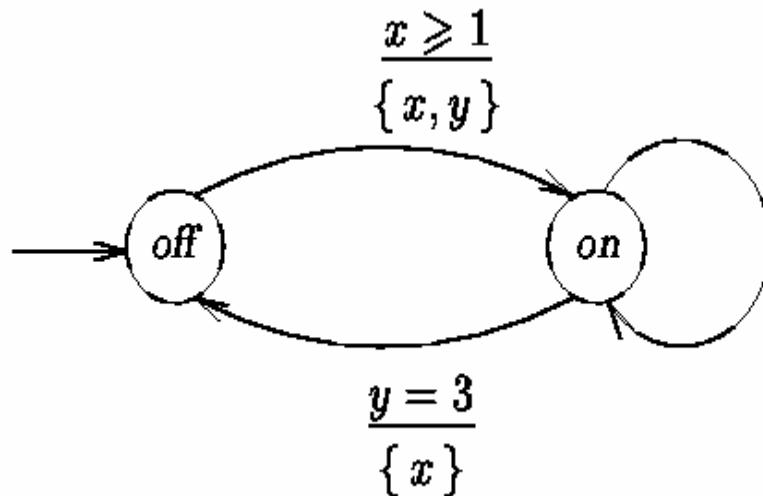
Reset Operation



An Example Region Graph



Modified light switch



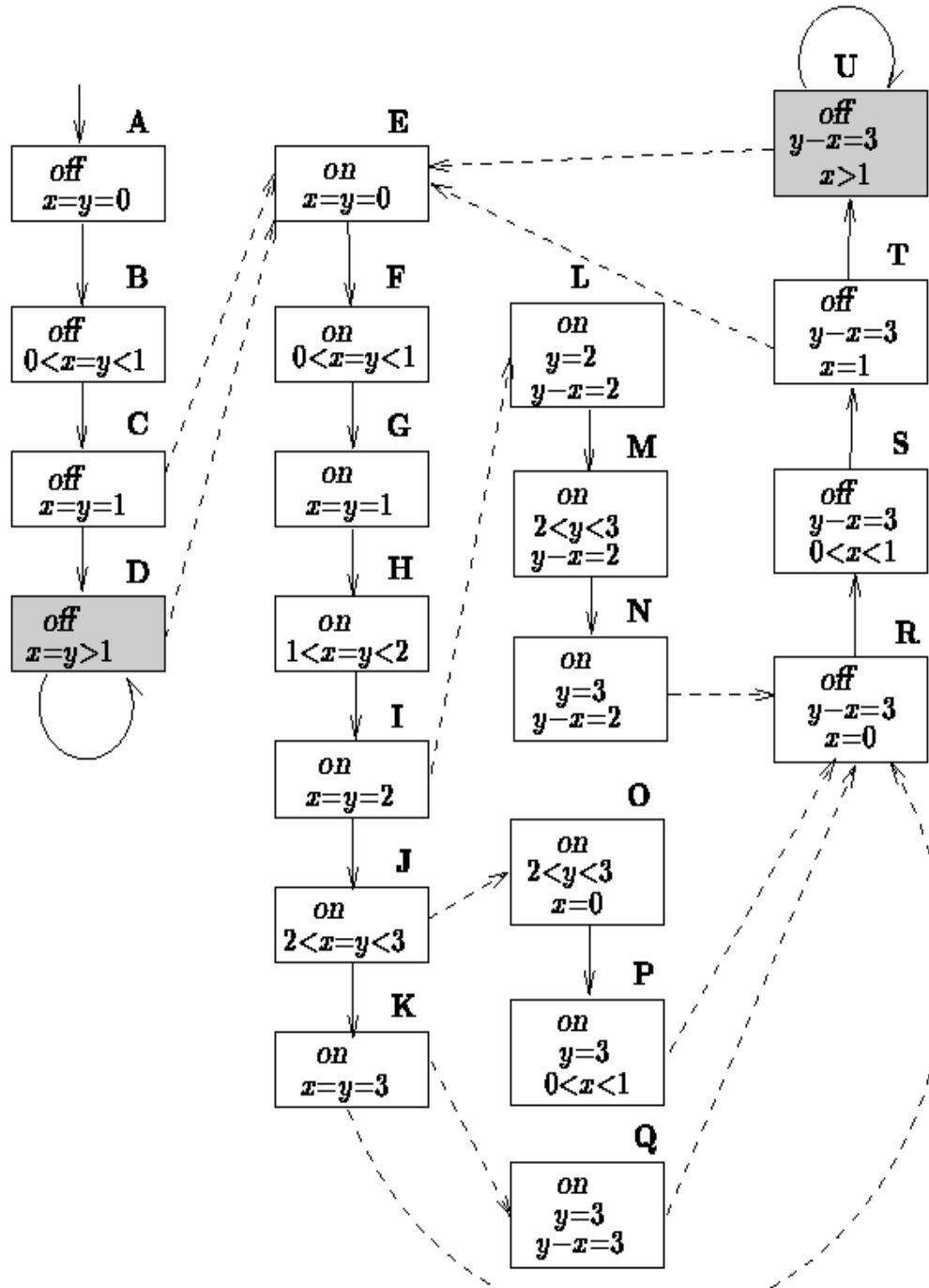
$$\frac{x \geq 2}{\{x\}}$$

$$\begin{aligned} \text{inv}(\text{off}) &= \text{true} \\ \text{inv}(\text{on}) &= y \leq 3 \end{aligned}$$

$$\text{AG}(x \leq y)$$

$$\text{AG}(\text{on} \Rightarrow \text{AFoff})$$

$$\text{AG}(\text{on} \Rightarrow \text{AF}_{\leq 9}\text{off})$$



Reachable part
of region graph

Properties

$$AG(x \leq y)$$

$$AG(on \Rightarrow AF_{off})$$

$$AG(on \Rightarrow AF_{\leq 9} off)$$

Finite Representation of Regions

Let Γ be a triple of the form:

$$\Gamma = \left\langle C_{>}, \mathcal{H}, [C_0, C_1, \dots, C_k] \right\rangle$$

where

- $C_{>}, C_0, C_1, \dots, C_k$ is a partitioning of C ($C_{>}$ and C_0 are allowed to be empty).
- $\mathcal{H} : (C \setminus C_{>}) \rightarrow \mathbb{N}$ with $\mathcal{H}(x) \leq c_x$.

Representation of Regions (cont)

$u \in \llbracket \Gamma \rrbracket$ if and only if

- $\forall x \in C_{>}. u(x) > c_x$;
- $\forall x \in C \setminus C_{>}. \lfloor u(x) \rfloor = \mathcal{H}(x)$;
- $\forall x \in C_0. fr(u(x)) = 0$;
- $\forall x, y \in C_i. fr(u(x)) = fr(u(y))$;
- $\forall x \in C_i. y \in C_j. fr(u(x)) < fr(u(y)) \Leftrightarrow i < j$;

$\llbracket \Gamma \rrbracket$ is a region, and any region is representable as a triplet.

Decidability & Complexity

Theorem

The transition relations $\xrightarrow{\delta}$ and \xrightarrow{a} between regions may be computed effectively using the triplet or the logical characterization of regions.

Theorem

Let A be a timed automaton with location l .
It is decidable whether $l \in Reach(A)$.

Theorem

Let A be a timed automaton with location l .
Deciding $l \in Reach(A)$ is PSPACE-complete.

Fundamental Results

- Reachability 😊 Alur, Dill
- Trace-inclusion Alur, Dill
 - Timed 😞 ; Untimed 😊
- Bisimulation
 - Timed 😊 Cerans ; Untimed 😊
- Model-checking 😊
 - TCTL, T_{mu} , L_{nu} , ...

Updatable Timed Automata

Patricia Bouyer, Catherine Dufourd,
Emmanuel Fleury, Antoine Petit

	Diagonal-free	W Diagonals
$x := c, x := y$	Pspace complete	Pspace complete
$x := x + 1$		Undecidable
$x := y + c$		
$x := x - 1$	Undecidable	
$x < c, x \leq c$	Pspace complete	Pspace complete
$x > c, x \geq c$		Undecidable
$x \sim y + c$		
$(y+c) < x < (y+d)$	Undecidable	
$y + c < x < z + d$		

With $\sim \in \{<, \leq, \geq, >\}$ and $c, d \in \mathbb{Q}_+$

	Diagonal-free	W Diagonals
$x := c, x := y$	TA-bisimilar	TA-bisimilar
$x := x + 1$		Turing
$x := y + c$		
$x < c, x \leq c$	TA $_{\epsilon}$	TA $_{\epsilon}$
$x > c, x \geq c$		Turing
$x \sim y + c$		
$(y+c) < x < (y+d)$		

With $\sim \in \{<, \leq, \geq, >\}$ and $c, d \in \mathbb{Q}_+$

TCTL: Timed Computational Tree Logic



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TCTL = CTL + Time

$$\phi ::= p \mid \alpha \mid \neg \phi \mid \phi \vee \phi \mid z \text{ in } \phi \mid E[\phi U \phi] \mid A[\phi U \phi]$$

$p \in AP$, atomic propositions,

$z \in D$, formula clocks,

α – constraints over formula clocks and automata clock

$z \text{ in } \phi$ – “freeze operator” introduces new
formula clock z

$E[\phi U \phi]$, $A[\phi U \phi]$ - like in CTL

No $EX \phi$

Derived Operators

$$\boxed{A[\phi U_{\leq 7} \psi]} = z \text{ in } A[(\phi \wedge z \leq 7) U \psi].$$

Along any path ϕ holds continuously until within 7 time units ψ becomes valid.

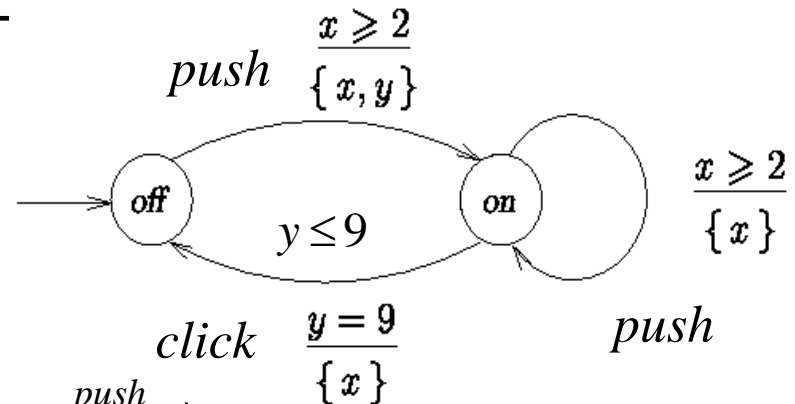
$$\boxed{EF_{<5} \phi} = z \text{ in } EF(z < 5 \wedge \phi)$$

The property ϕ becomes valid within 5 time units.

Paths

A *path* is an infinite sequence $s_0 a_0 s_1 a_1 s_2 a_2 \dots$ of states alternated by transition labels such that $s_i \xrightarrow{a_i} s_{i+1}$ for all $i \geq 0$.

Example:



$$(off, x = y = 0) \xrightarrow{3.5} (off, x = y = 3.5) \xrightarrow{push} \dots$$

$$(on, x = y = 0) \xrightarrow{\pi} (on, x = y = \pi) \xrightarrow{push} \dots$$

$$(on, x = 0, y = \pi) \xrightarrow{3} (on, x = 3, y = \pi + 3) \xrightarrow{9 - (\pi + 3)} \dots$$

$$(on, x = 9 - (\pi + 3), y = 9) \xrightarrow{click} (off, x = 0, y = 9) \dots$$

Elapsed time in path

$$\Delta(\sigma, 0) = 0$$

$$\Delta(\sigma, i+1) = \Delta(\sigma, i) + \begin{cases} 0 & \text{if } a_i = * \\ a_i & \text{if } a_i \in \mathbb{R}^+. \end{cases}$$

Example:

$$\begin{aligned} \sigma = & (off, x = y = 0) \xrightarrow{3.5} (off, x = y = 3.5) \xrightarrow{push} \\ & (on, x = y = 0) \xrightarrow{\pi} (on, x = y = \pi) \xrightarrow{push} \\ & (on, x = 0, y = \pi) \xrightarrow{3} (on, x = 3, y = \pi + 3) \xrightarrow{9-(\pi+3)} \\ & (on, x = 9 - (\pi + 3), y = 9) \xrightarrow{click} (off, x = 0, y = 9) \dots \end{aligned}$$

$$\Delta(\sigma, 1) = 3.5, \Delta(\sigma, 6) = 3.5 + 9 = 12.5$$

TCTL Semantics

$s, w \models p$	iff $p \in \text{Label}(s)$
$s, w \models \alpha$	iff $v \cup w \models \alpha$
$s, w \models \neg \phi$	iff $\neg(s, w \models \phi)$
$s, w \models \phi \vee \psi$	iff $(s, w \models \phi) \vee (s, w \models \psi)$
$s, w \models z \text{ in } \phi$	iff $s, \text{reset } z \text{ in } w \models \phi$
$s, w \models E[\phi U \psi]$	iff $\exists \sigma \in P_M^\infty(s). \exists (i, d) \in \text{Pos}(\sigma).$ $(\sigma(i, d), w + \Delta(\sigma, i) \models \psi \wedge$ $(\forall (j, d') \ll (i, d). \sigma(j, d'), w + \Delta(\sigma, j) \models \phi \vee \psi))$
$s, w \models A[\phi U \psi]$	iff $\forall \sigma \in P_M^\infty(s). \exists (i, d) \in \text{Pos}(\sigma).$ $((\sigma(i, d), w + \Delta(\sigma, i)) \models \psi \wedge$ $(\forall (j, d') \ll (i, d). (\sigma(j, d'), w + \Delta(\sigma, j)) \models \phi \vee \psi)).$

s - location

w - formula clock valuation

$P_M^\infty(s)$ - set of paths from s

$\text{Pos}(\sigma)$ - positions in σ

$\Delta(\sigma, i)$ - elapsed time

$(i, d) \ll (i', d')$ iff $(i < j)$ or $((i = j) \text{ and } (d < d'))$

Timeliness Properties

$$\text{AG} [\text{send}(m) \Rightarrow \text{AF}_{<5} \text{receive}(r_m)]$$

receive(m) occurs within 5 time units after *send(m)*

$$\text{EG} [\text{send}(m) \Rightarrow \text{AF}_{=11} \text{receive}(r_m)]$$

receive(m) occurs exactly 11 time units after *send(m)*

$$\text{AG} [\text{AF}_{=25} \text{putbox}]$$

putbox occurs periodically (exactly) every 25 time units
(note: other *putbox*'s may occur in between)

The UPPAAL Verification Engine



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Overview

- Zones and DBMs
- Minimal Constraint Form
- Clock Difference Diagrams

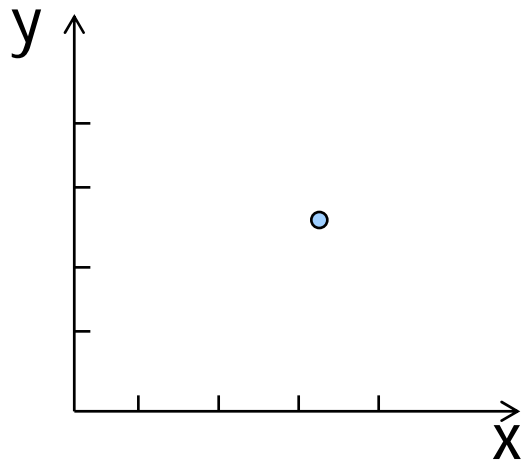
- Distributed UPPAAL [CAV2000, STTT2004]
- Unification & Sharing [FTRTFT2002, SPIN2003]
- Acceleration [FORMATS2002]
- Static Guard Analysis [TACAS2003, TACAS2004]
- Storage-Strategies [CAV2003]

Zones

From infinite to finite

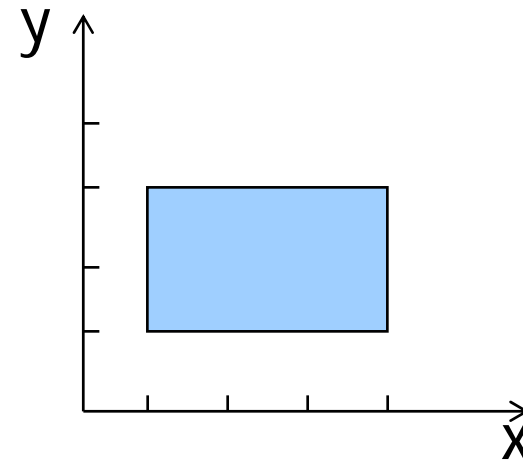
State

$(n, x=3.2, y=2.5)$



Symbolic state (set)

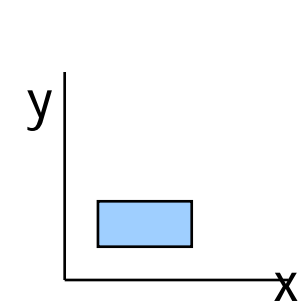
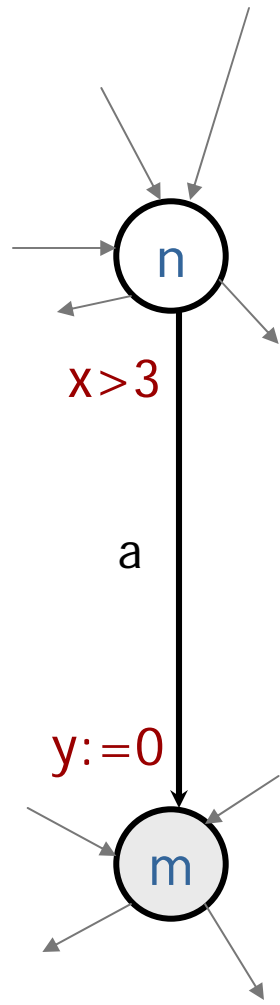
$(n, 1 \leq x \leq 4, 1 \leq y \leq 3)$



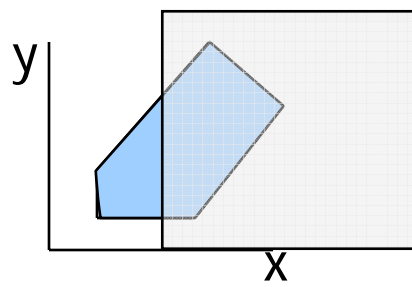
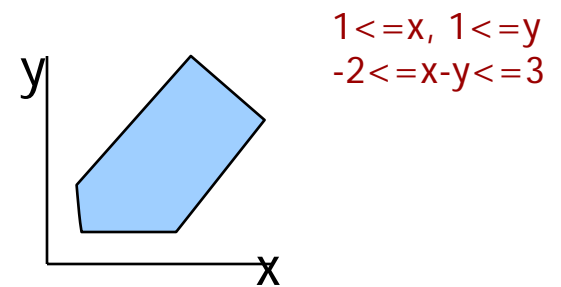
Zone:

conjunction of
 $x-y \leq n, x \leq n$

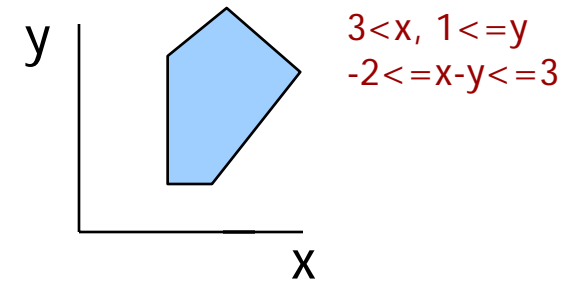
Symbolic Transitions



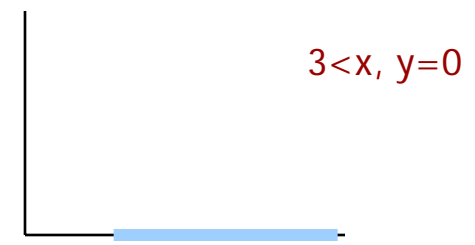
delays to



conjuncts to



projects to



Thus $(n, 1 \leq x \leq 4, 1 \leq y \leq 3) = a \Rightarrow (m, 3 < x, y = 0)$

Zones = Conjunctive Constraints

- A zone Z is a conjunctive formula:

$$g_1 \ \& \ g_2 \ \& \ \dots \ \& \ g_n$$

where g_i is a clock constraint $x_i \sim b_i$ or $x_i - x_j \sim b_{ij}$

- Use a zero-clock x_0 (constant 0)

- A zone can be re-written as a set:

$$\{x_i - x_j \sim b_{ij} \mid \sim \text{ is } < \text{ or } \leq, i, j \leq n\}$$

- This can be represented as a matrix, DBM (Difference Bound Matrices)

Operations on Zones

- Future delay $Z\uparrow$:

$$[Z\uparrow] = \{u+d \mid d \in \mathbb{R}, u \in [Z]\}$$

- Past delay $Z\downarrow$:

$$[Z\downarrow] = \{u \mid u+d \in [Z] \text{ for some } d \in \mathbb{R}\}$$

- Reset: $\{x\}Z$ or $Z(x:=0)$

$$[\{x\}Z] = \{u[0/x] \mid u \in [Z]\}$$

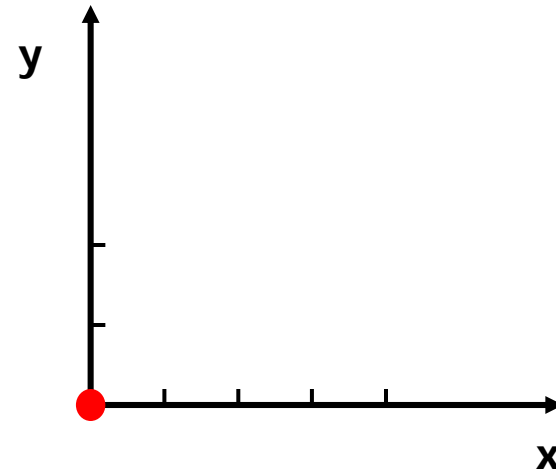
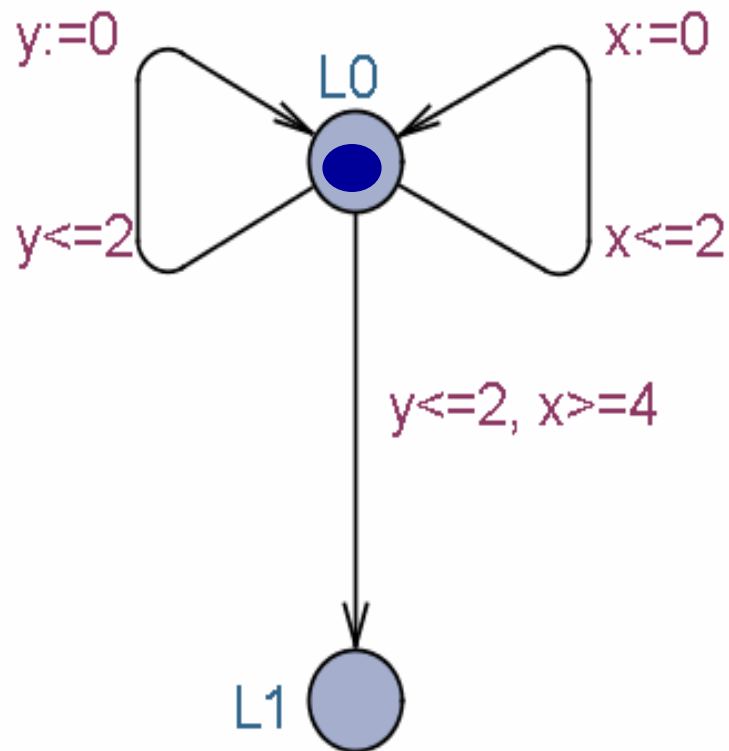
- Conjunction

$$[Z\&g] = [Z] \cap [g]$$

THEOREM

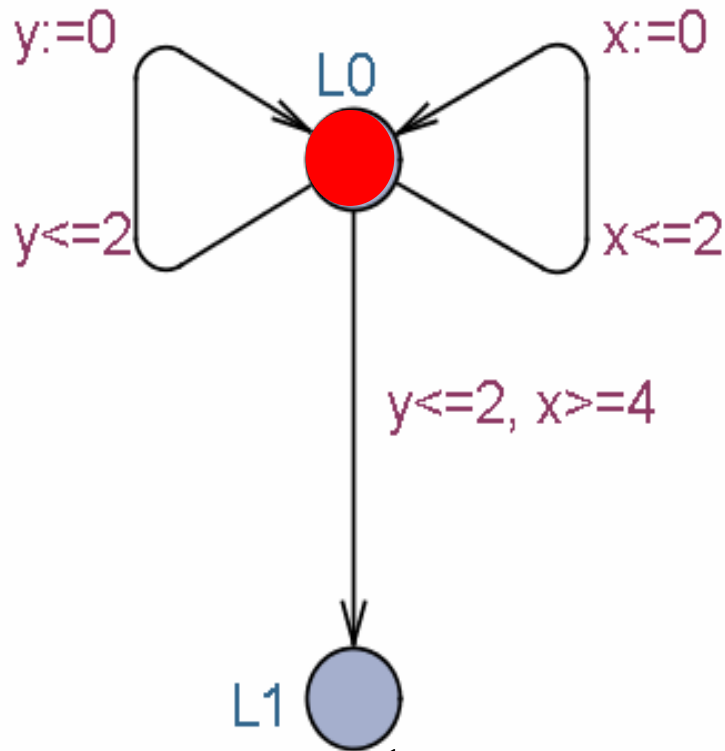
- The set of zones is closed under all constraint operations.
- That is, the result of the operations on a zone is a zone.
- That is, there will be a zone (a finite object i.e a zone/constraints) to represent the sets: $[Z\uparrow]$, $[Z\downarrow]$, $[\{x\}Z]$, $[Z\&g]$.

Symbolic Exploration

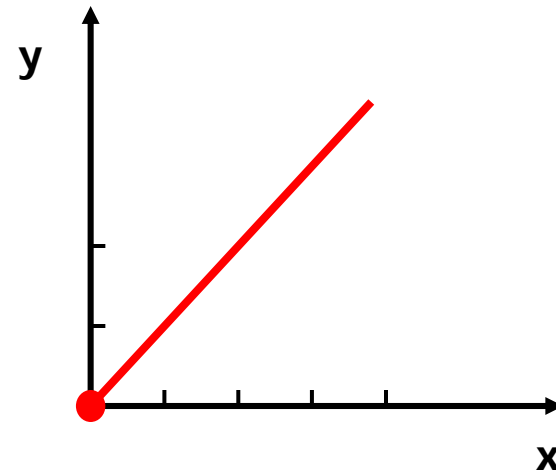


Reachable?

Symbolic Exploration

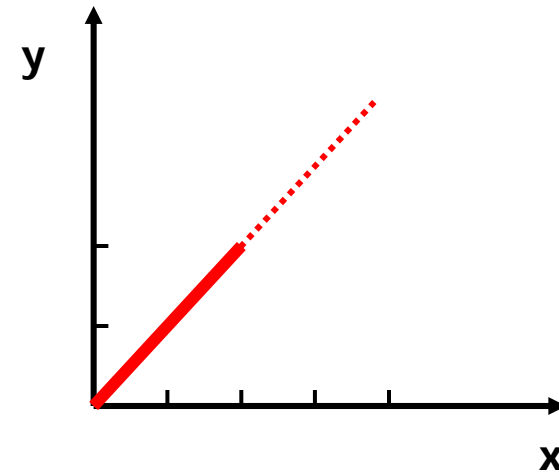
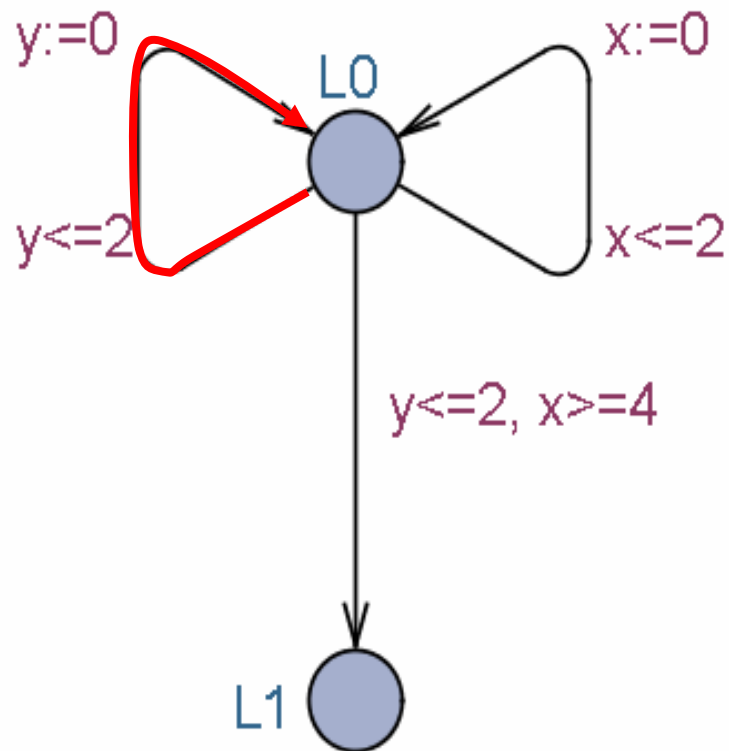


Reachable?



Delay

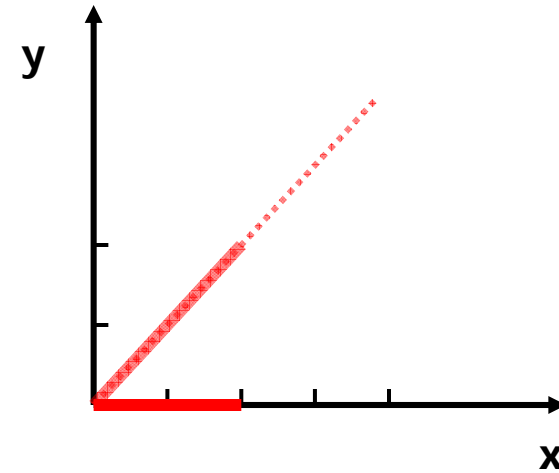
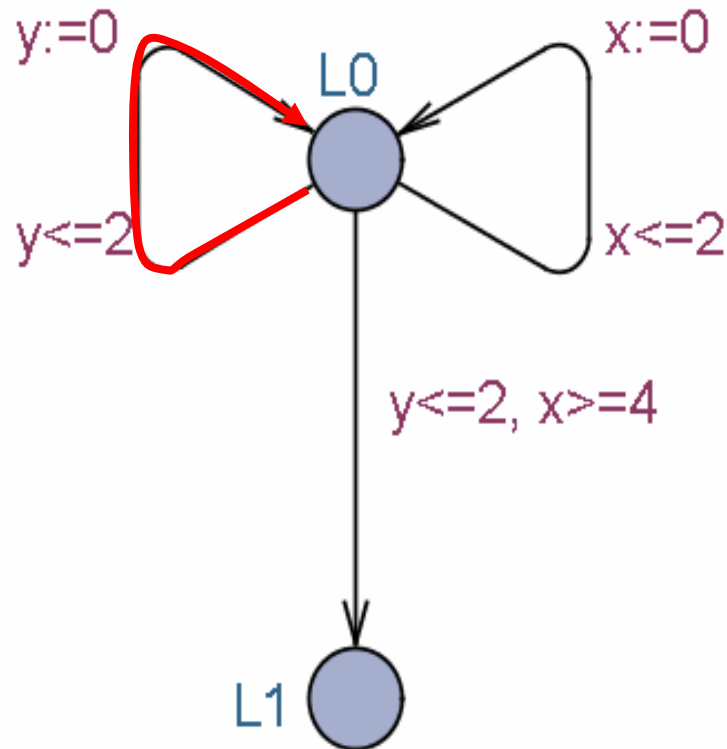
Symbolic Exploration



Left

Reachable?

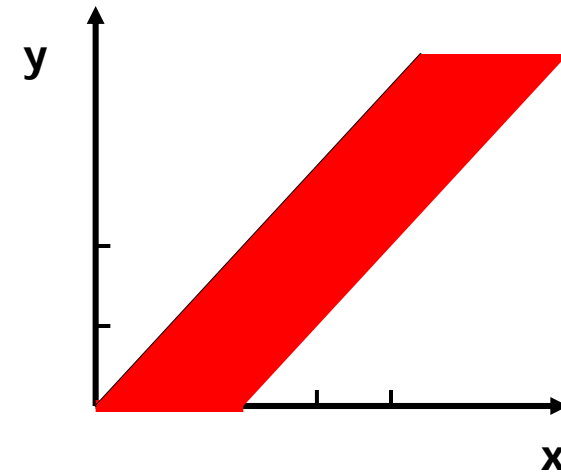
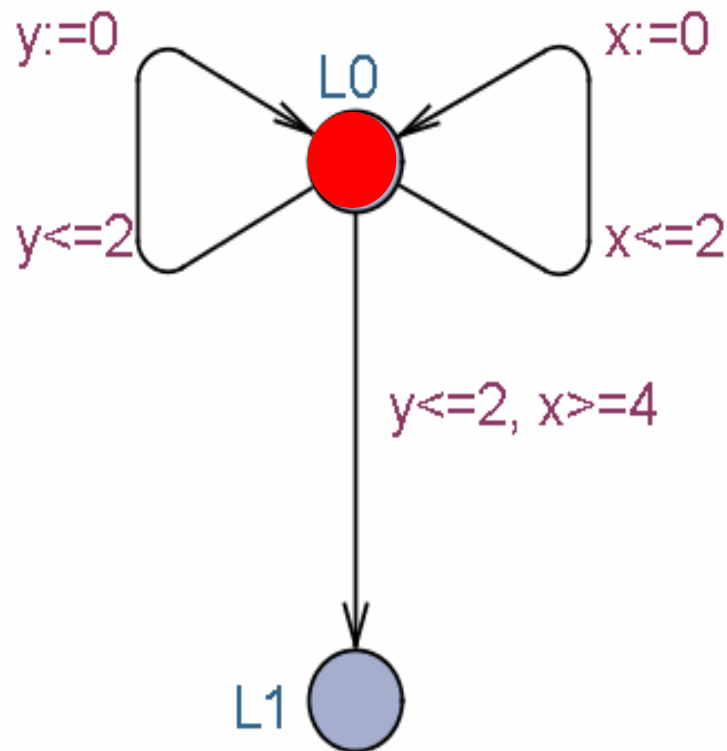
Symbolic Exploration



Left

Reachable?

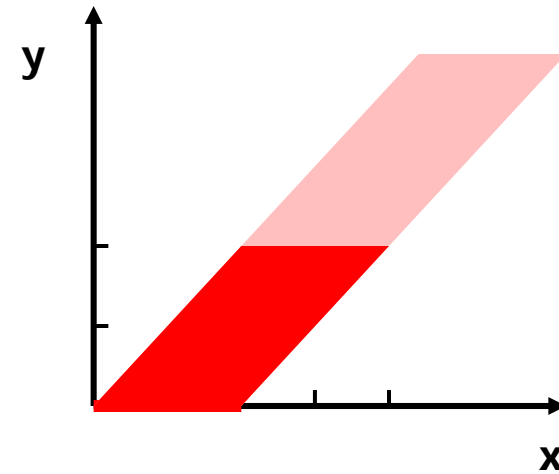
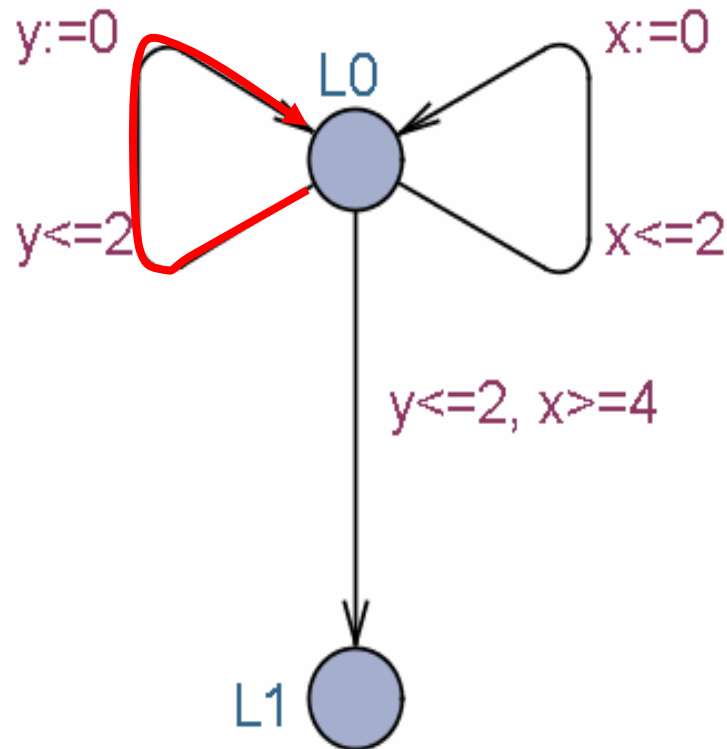
Symbolic Exploration



Delay

Reachable?

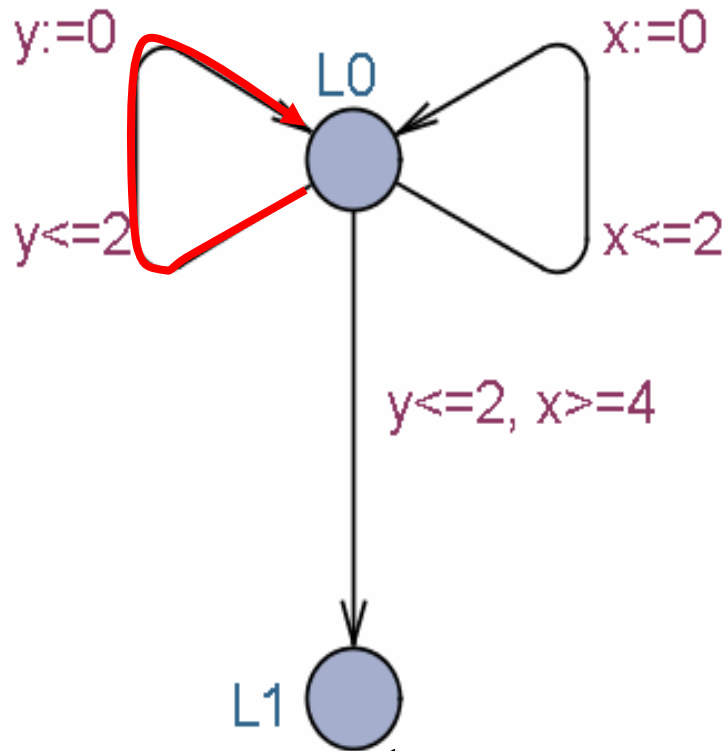
Symbolic Exploration



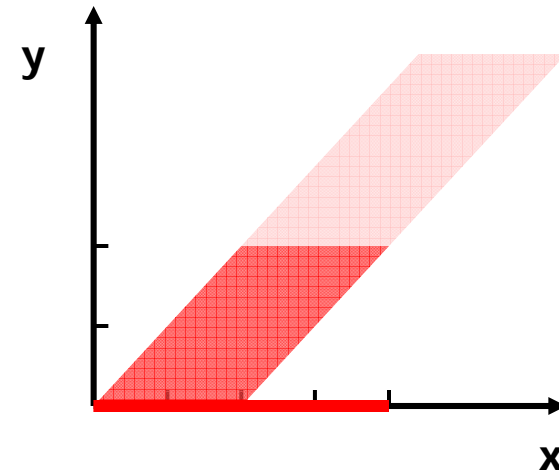
Left

Reachable?

Symbolic Exploration

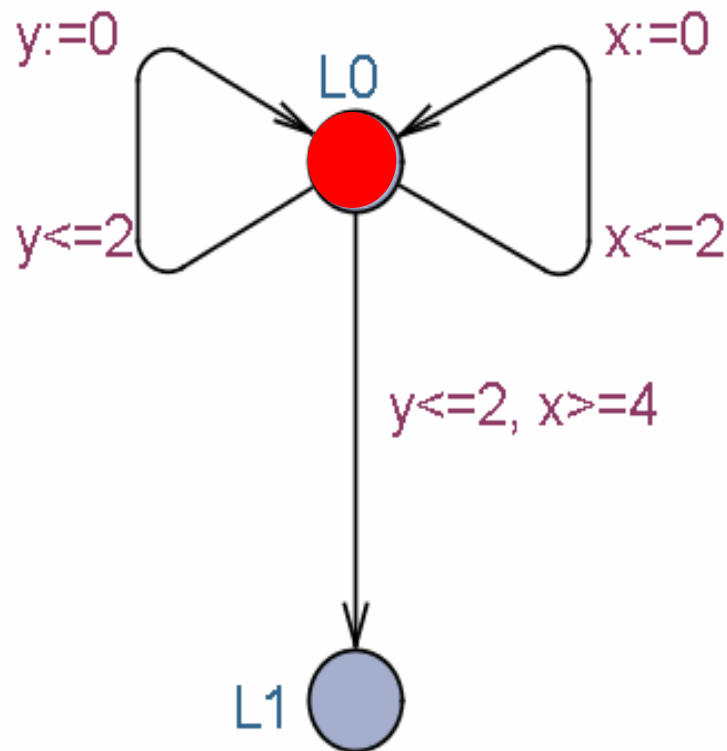


Reachable?

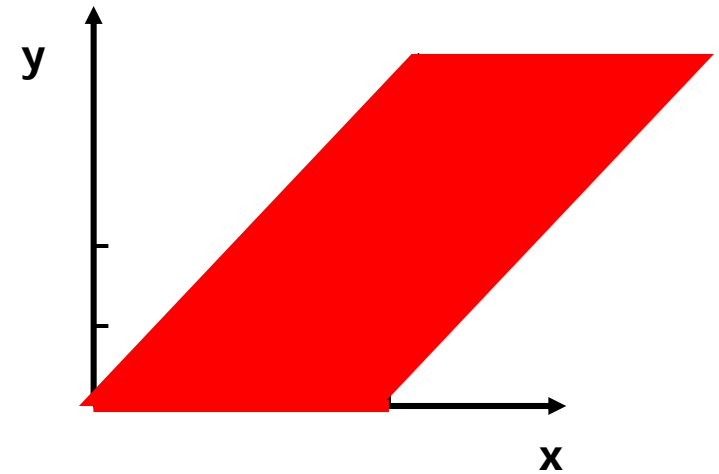


Left

Symbolic Exploration

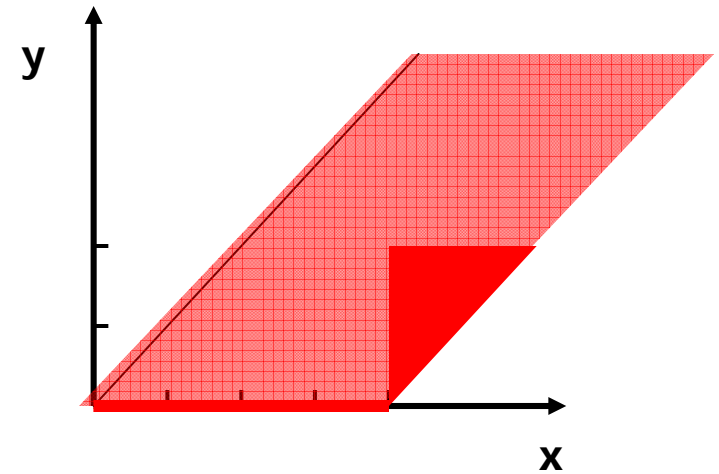
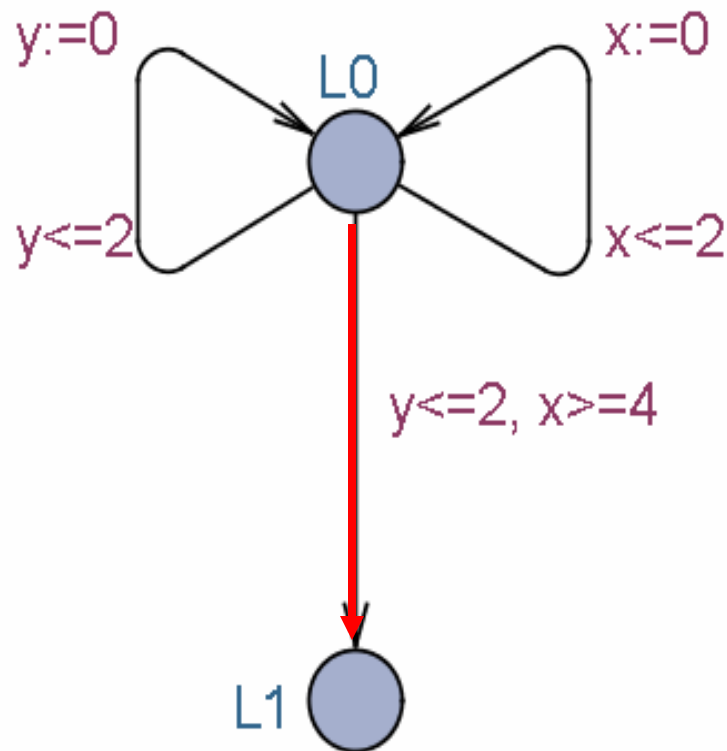


Reachable?



Delay

Symbolic Exploration

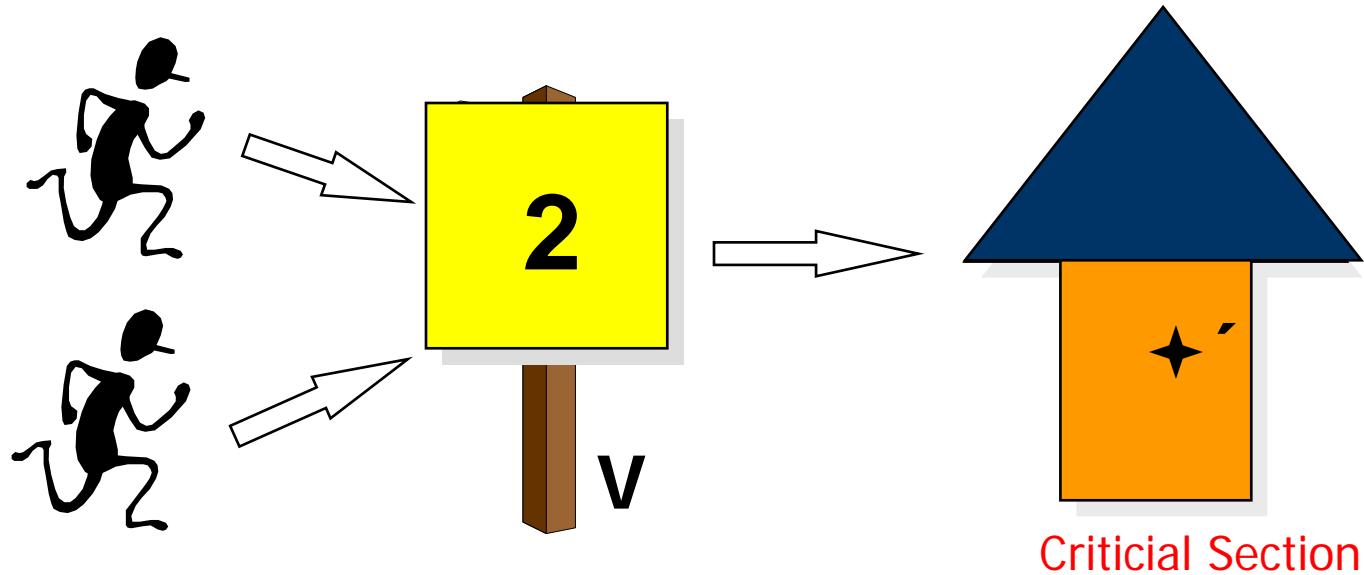


Down

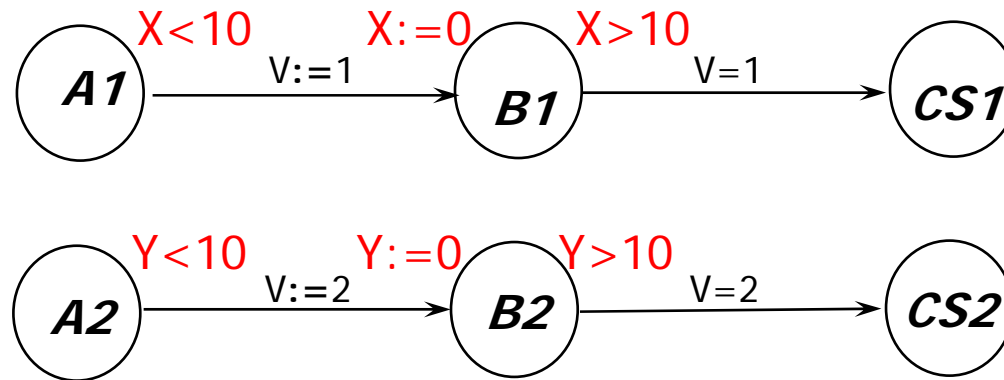
Reachable?

Fischer's Protocol

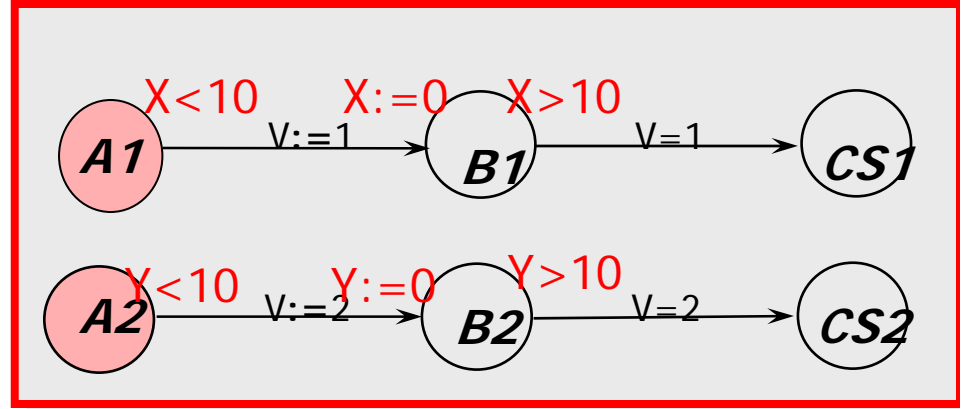
analysis using zones



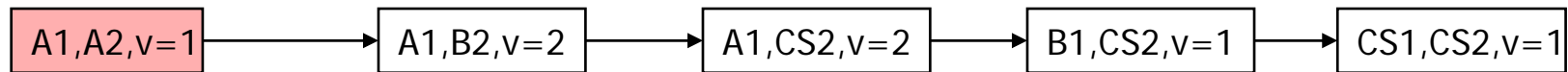
Init
 $V=1$



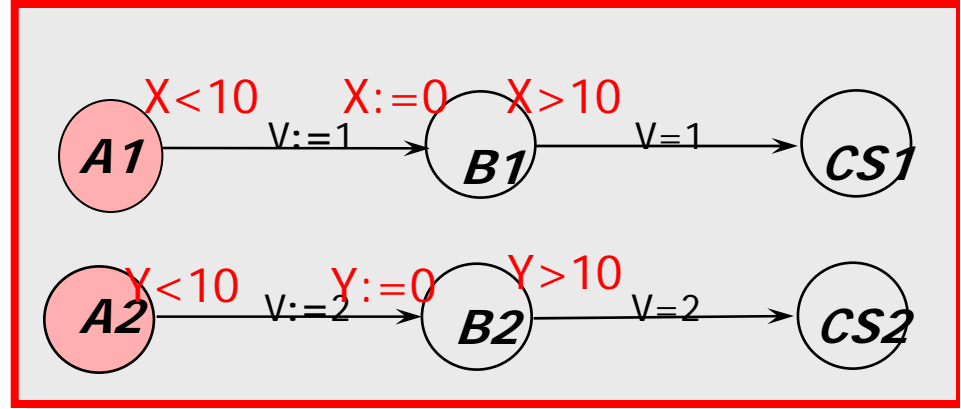
Fischers cont.



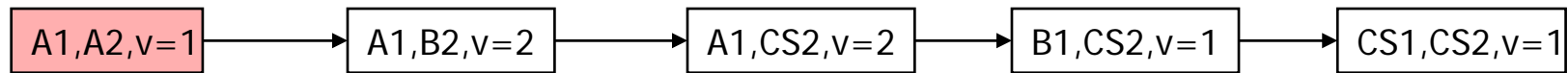
Untimed case



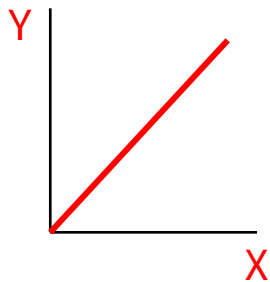
Fischers cont.



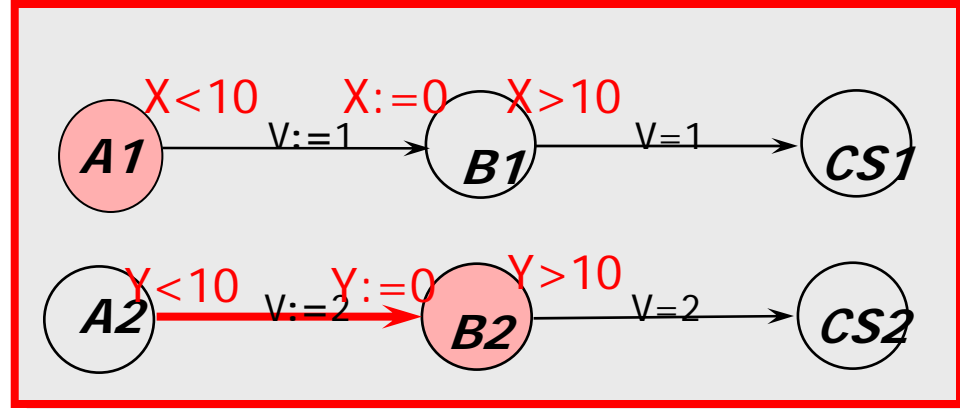
Untimed case



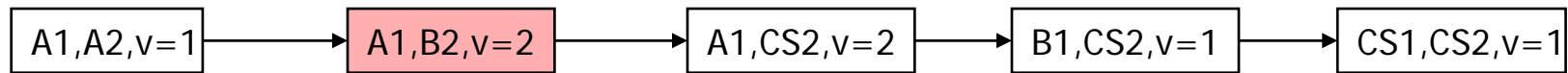
Taking time into account



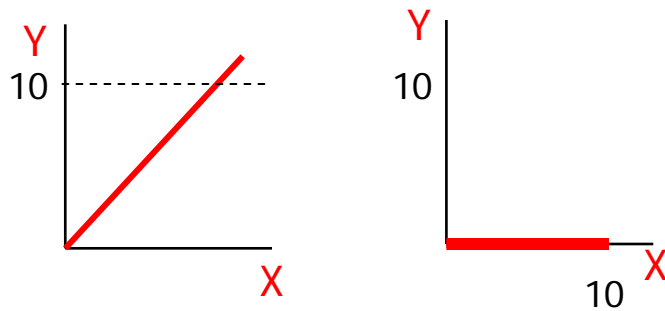
Fischers cont.



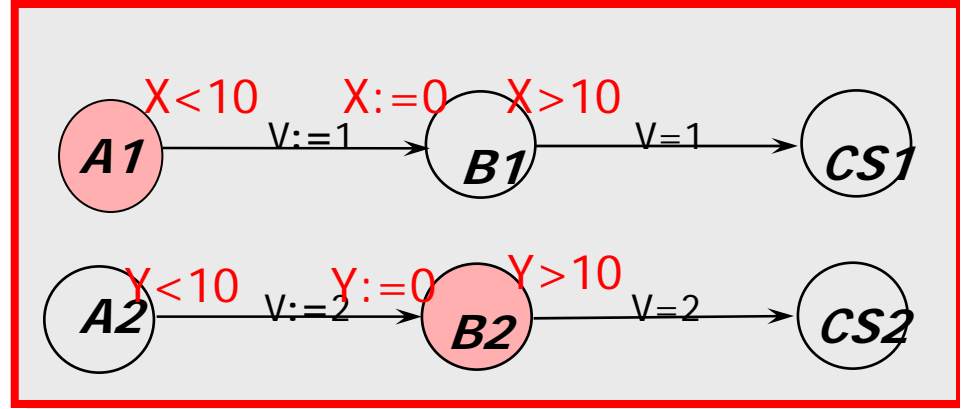
Untimed case



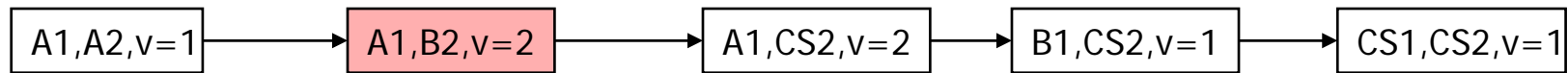
Taking time into account



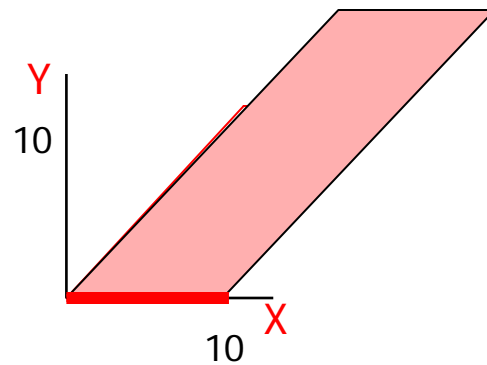
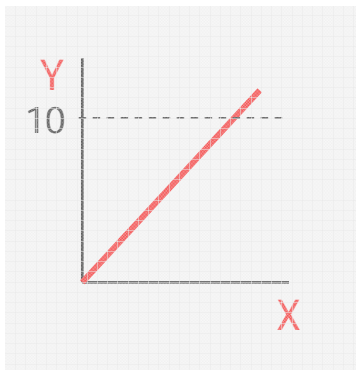
Fischers cont.



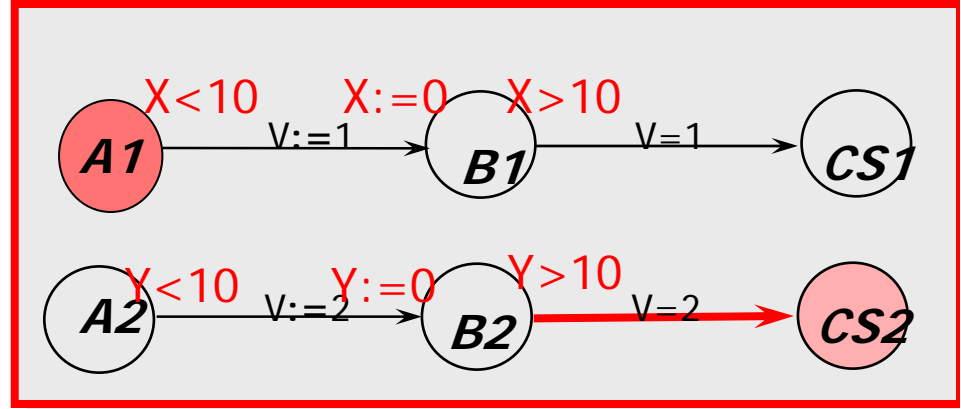
Untimed case



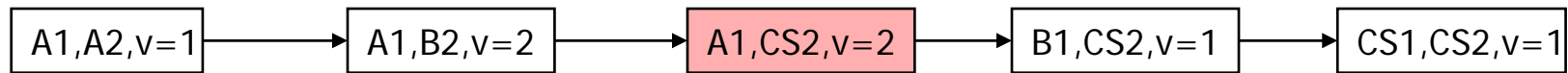
Taking time into account



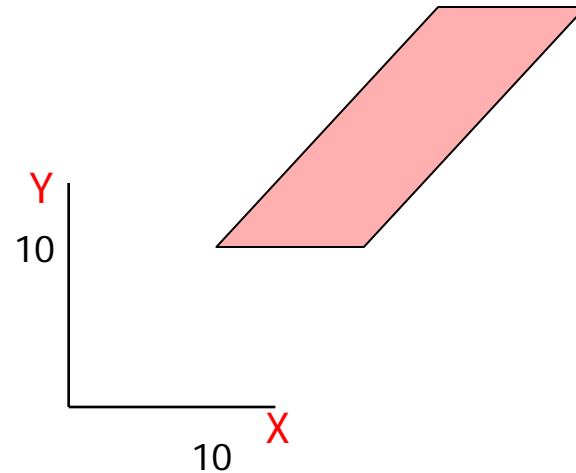
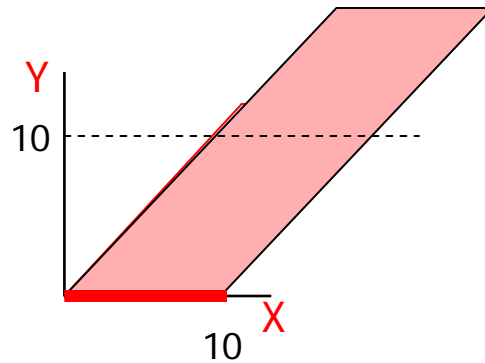
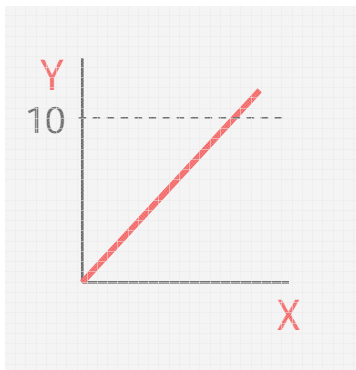
Fischers cont.



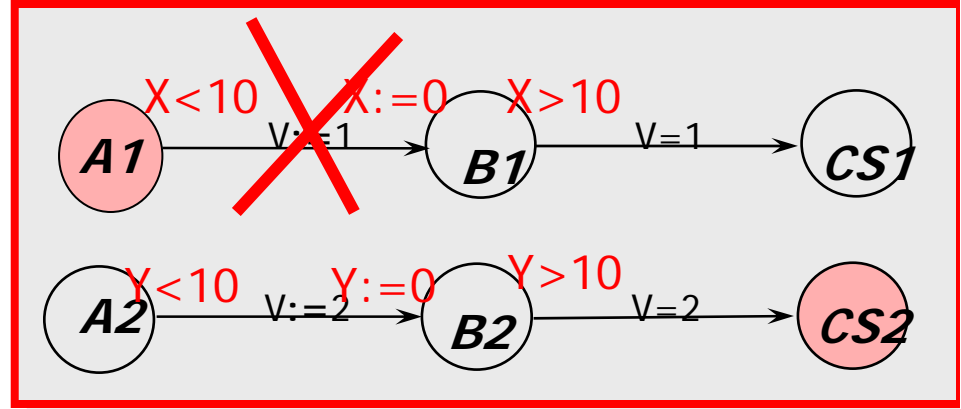
Untimed case



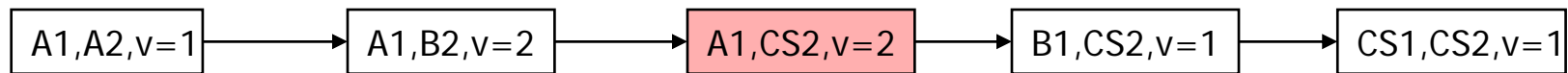
Taking time into account



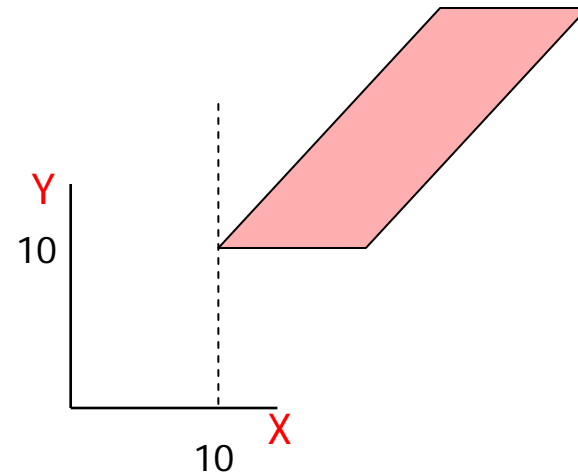
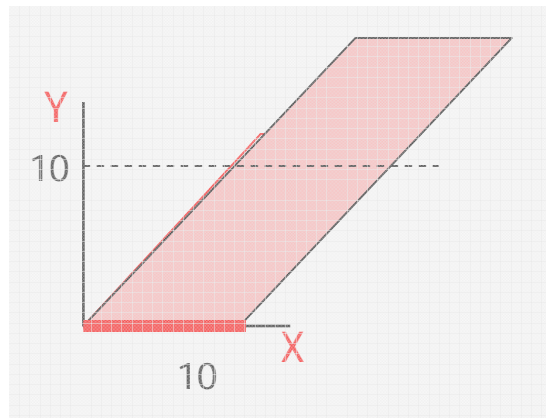
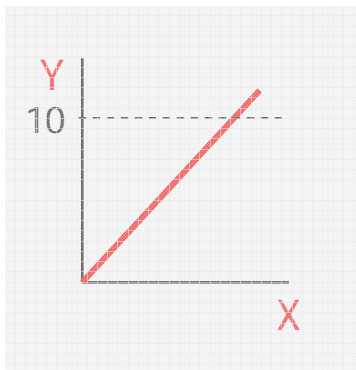
Fischers cont.



Untimed case

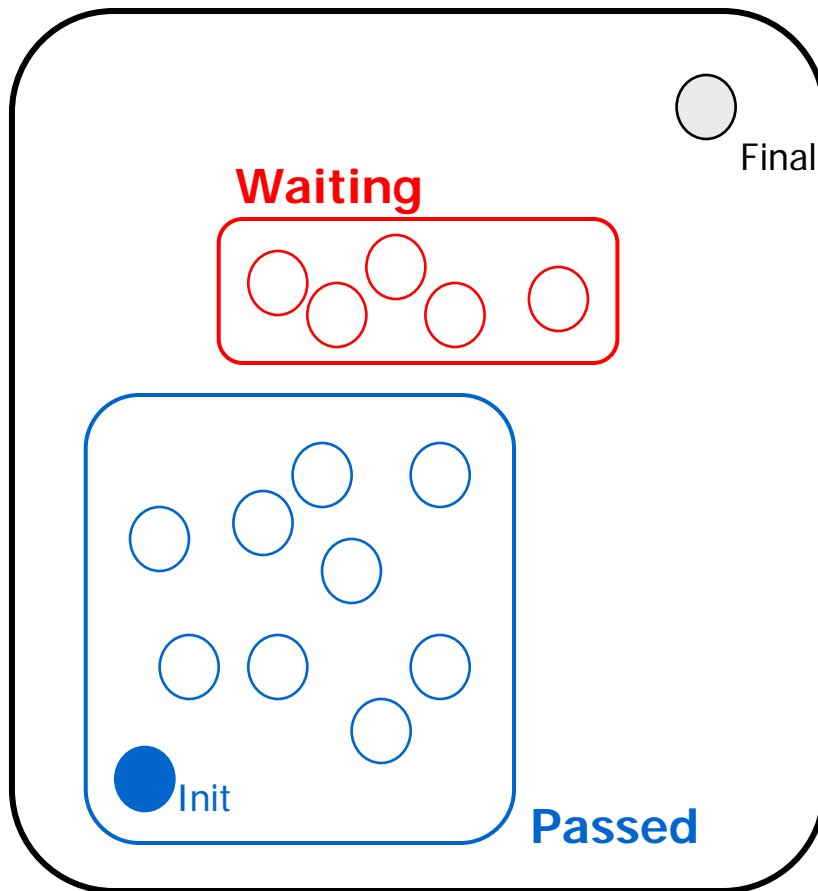


Taking time into account



Forward Reachability

Init -> Final ?



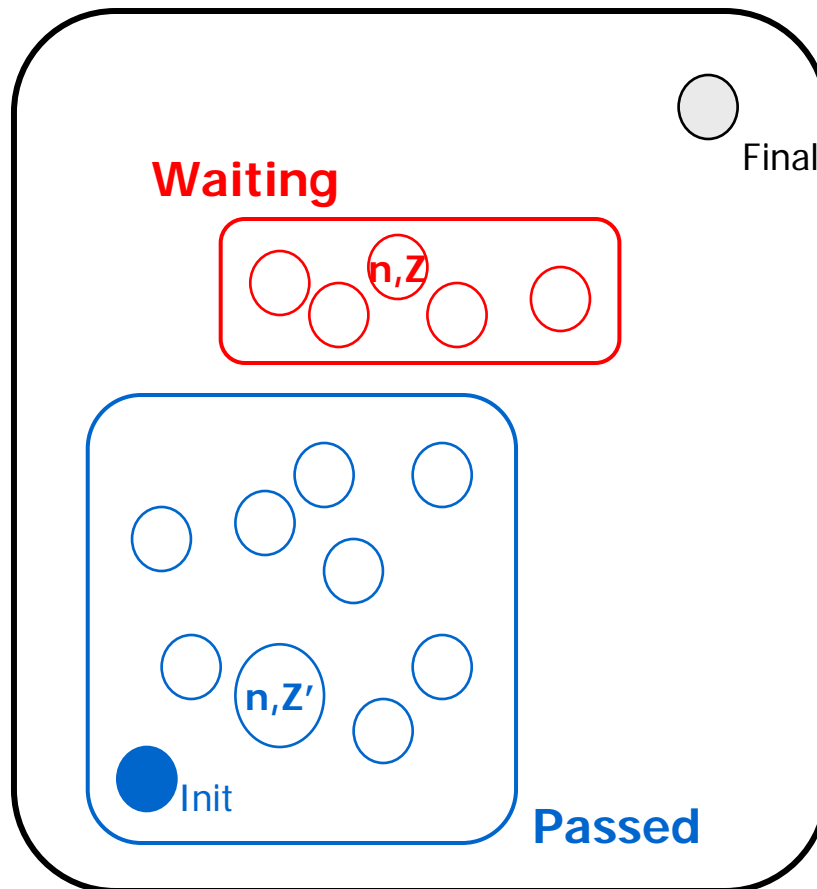
INITIAL Passed := \emptyset ;
 Waiting := $\{(n0, Z0)\}$

REPEAT

UNTIL Waiting = \emptyset
 or
 Final is in **Waiting**

Forward Reachability

Init \rightarrow Final ?



INITIAL **Passed** := \emptyset ;
Waiting := $\{(n_0, Z_0)\}$

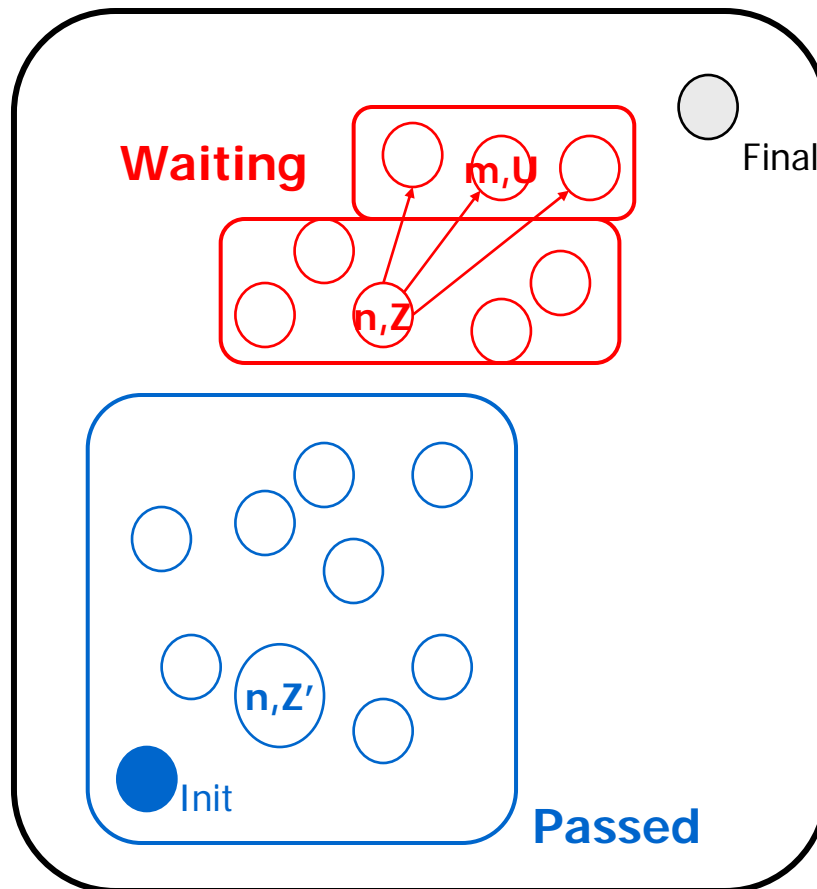
REPEAT

- pick (n, Z) in **Waiting**
- **if** for some $Z' \supseteq Z$
 (n, Z') in **Passed** then **STOP**

UNTIL **Waiting** = \emptyset
or
Final is in **Waiting**

Forward Reachability

Init \rightarrow Final ?



INITIAL $Passed := \emptyset;$
Waiting $:= \{(n_0, Z_0)\}$

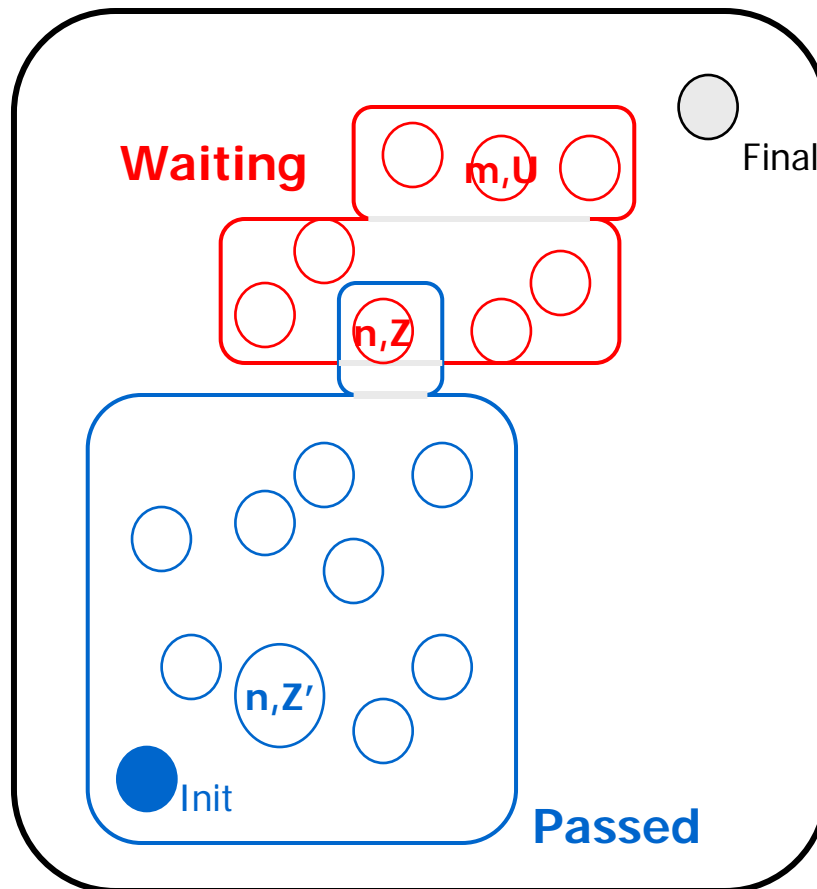
REPEAT

- pick (n, Z) in **Waiting**
- **if** for some $Z' \supseteq Z$
 (n, Z') in **Passed** then **STOP**
- **else** /explore/ add
 $\{ (m, U) : (n, Z) \Rightarrow (m, U) \}$
to **Waiting**;

UNTIL **Waiting** $= \emptyset$
or
Final is in **Waiting**

Forward Reachability

Init -> Final ?



INITIAL **Passed** := \emptyset ;
Waiting := $\{(n_0, Z_0)\}$

REPEAT

- pick (n, Z) in **Waiting**
- **if** for some $Z' \supseteq Z$
 (n, Z') in **Passed** then **STOP**
- **else** /explore/ add
 $\{(m, U) : (n, Z) \Rightarrow (m, U)\}$
to **Waiting**;
Add (n, Z) to **Passed**

UNTIL **Waiting** = \emptyset
or
Final is in **Waiting**

Canonical Datastructures for Zones

Difference Bounded Matrices

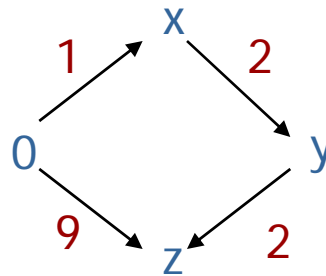
Bellman 1958, Dill 1989

Inclusion

D1

$$\begin{array}{l} x \leq 1 \\ y - x \leq 2 \\ z - y \leq 2 \\ z \leq 9 \end{array}$$

Graph

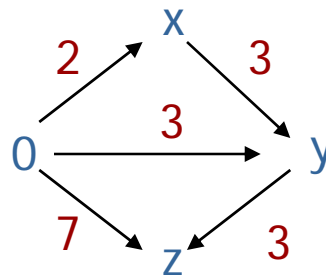


? \subseteq ?

D2

$$\begin{array}{l} x \leq 2 \\ y - x \leq 3 \\ y \leq 3 \\ z - y \leq 3 \\ z \leq 7 \end{array}$$

Graph



Canonical Datastructures for Zones

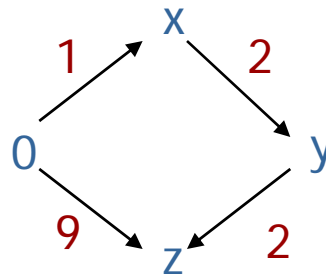
Difference Bounded Matrices

Inclusion

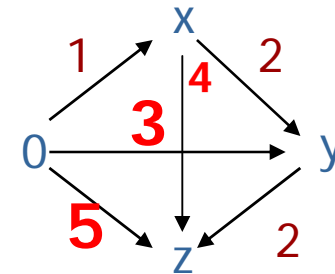
D1

$$\begin{aligned} x &\leq 1 \\ y - x &\leq 2 \\ z - y &\leq 2 \\ z &\leq 9 \end{aligned}$$

Graph



Shortest
Path
Closure

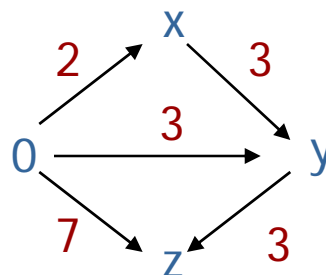


$$? \subseteq ?$$

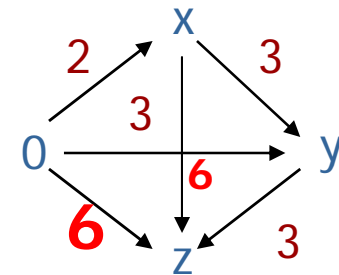
D2

$$\begin{aligned} x &\leq 2 \\ y - x &\leq 3 \\ y &\leq 3 \\ z - y &\leq 3 \\ z &\leq 7 \end{aligned}$$

Graph



Shortest
Path
Closure



Canonical Datastructures for Zones

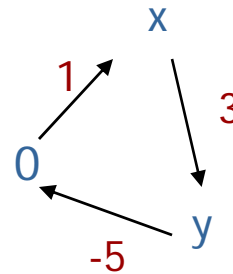
Difference Bounded Matrices

Emptiness

D

$$\begin{array}{l} x \leq 1 \\ y \geq 5 \\ y - x \leq 3 \end{array}$$

Graph



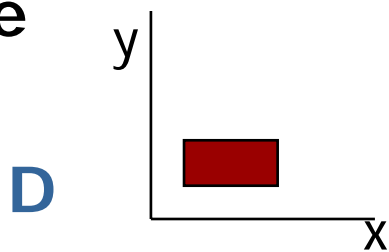
Negative Cycle
iff
empty solution set

Compact

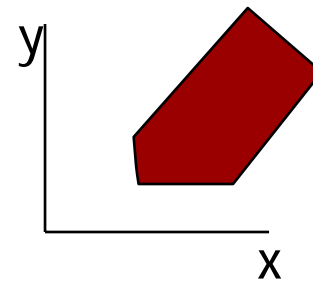
Canonical Datastructures for Zones

Difference Bounded Matrices

Future

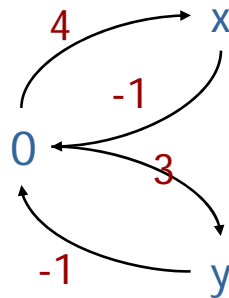


$$\begin{aligned} 1 &\leq x \leq 4 \\ 1 &\leq y \leq 3 \end{aligned}$$

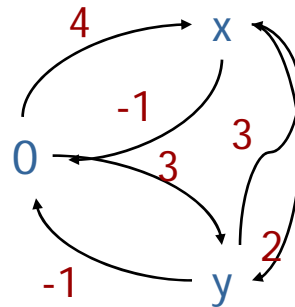


Future D

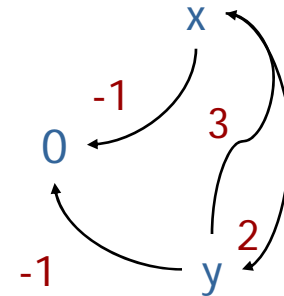
$$\begin{aligned} 1 &\leq x, 1 \leq y \\ -2 &\leq x - y \leq 3 \end{aligned}$$



Shortest Path Closure



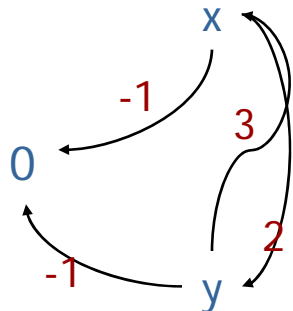
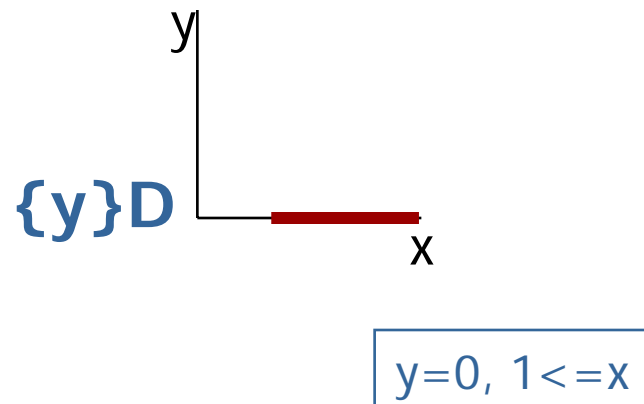
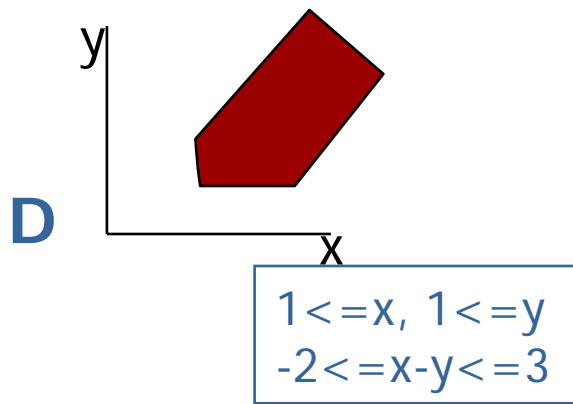
Remove upper bounds on clocks



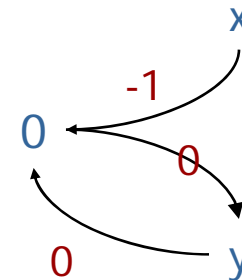
Canonical Datastructures for Zones

Difference Bounded Matrices

Reset



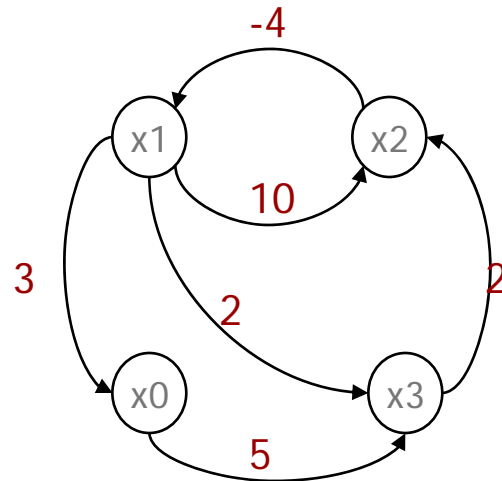
Remove all
bounds
involving y
and set y to 0



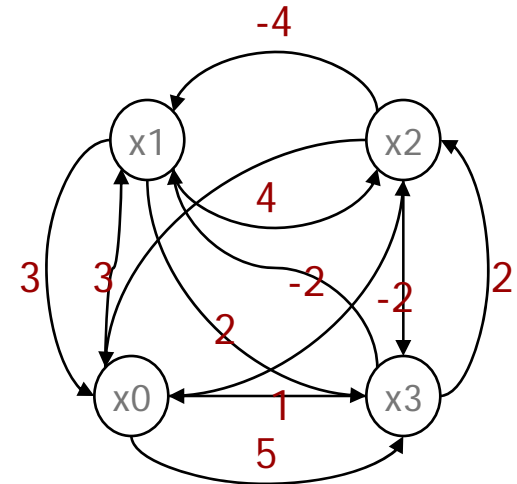
Canonical Datastructures for Zones

Difference Bounded Matrices

$x_1 - x_2 \leq 4$
 $x_2 - x_1 \leq 10$
 $x_3 - x_1 \leq 2$
 $x_2 - x_3 \leq 2$
 $x_0 - x_1 \leq 3$
 $x_3 - x_0 \leq 5$



**Shortest
Path
Closure
 $O(n^3)$**

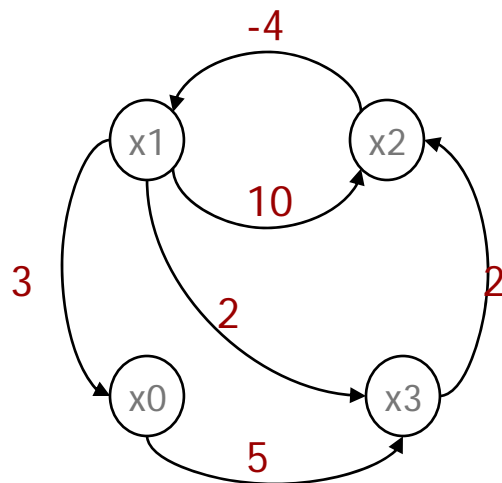


Canonical Datastructures for Zones

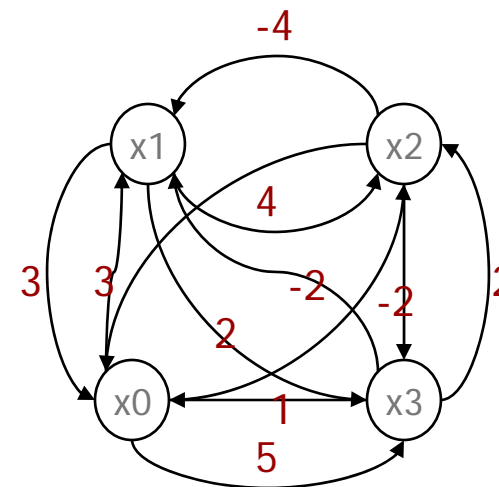
Minimal Constraint Form

RTSS 1997

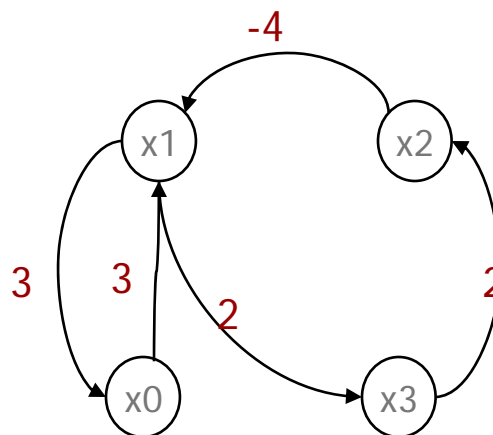
- $x1 - x2 \leq 4$
- $x2 - x1 \leq 10$
- $x3 - x1 \leq 2$
- $x2 - x3 \leq 2$
- $x0 - x1 \leq 3$
- $x3 - x0 \leq 5$



Shortest Path Closure
 $O(n^3)$

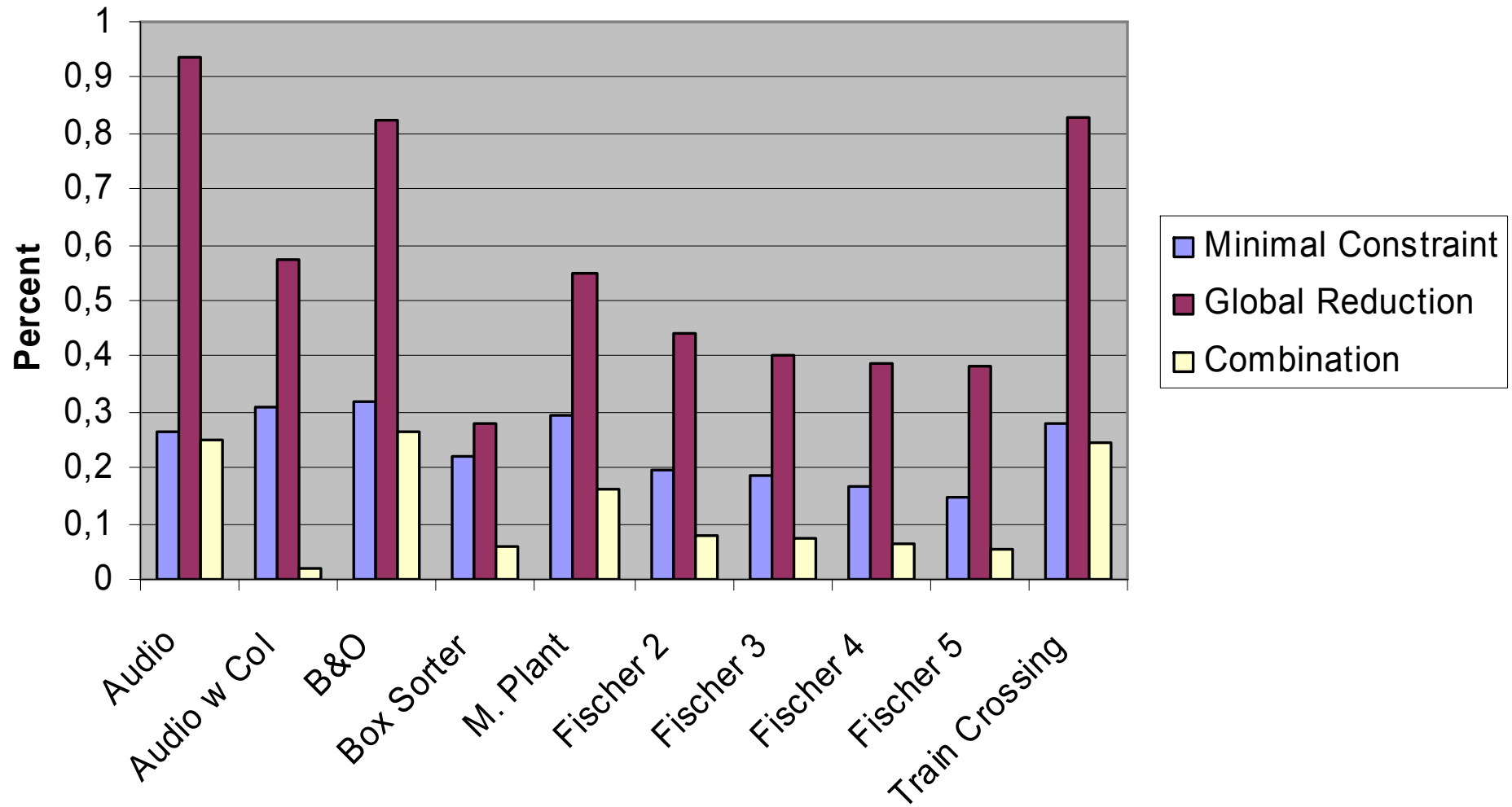


Shortest Path Reduction
 $O(n^3)$

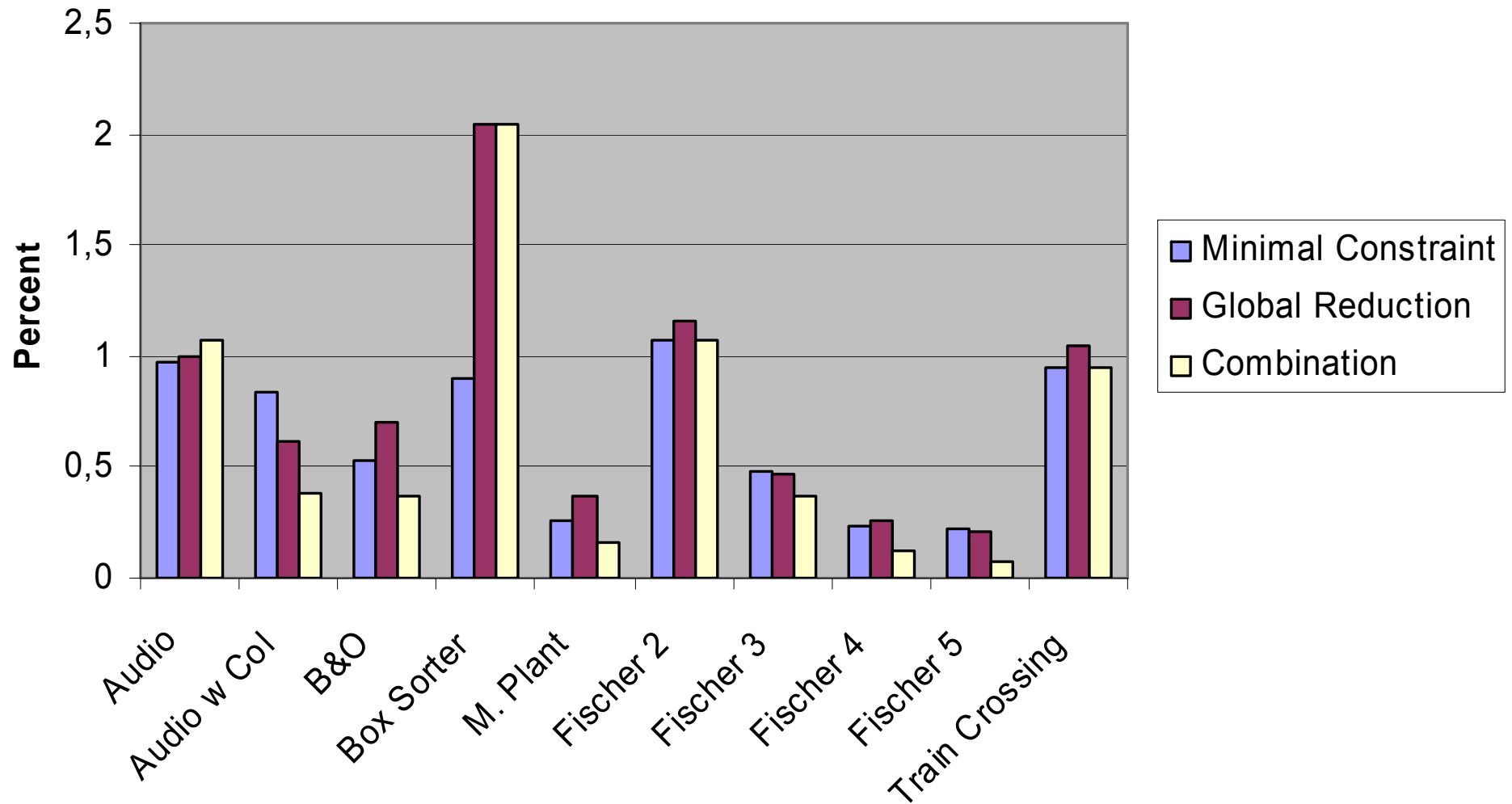


Space worst $O(n^2)$
practice $O(n)$

SPACE PERFORMANCE



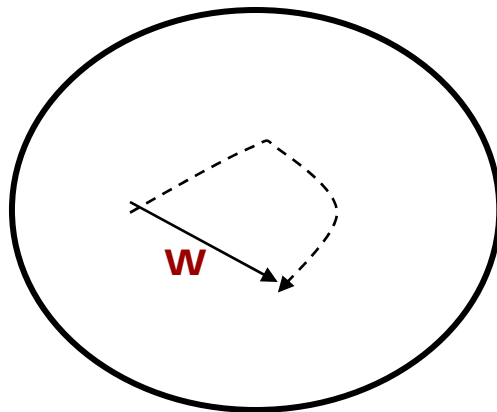
TIME PERFORMANCE



Shortest Path Reduction

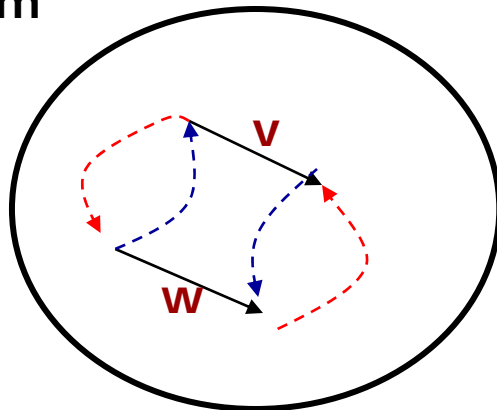
1st attempt

Idea



An edge is **REDUNDANT** if there exists an alternative path of no greater weight
 THUS **Remove all redundant edges!**

Problem

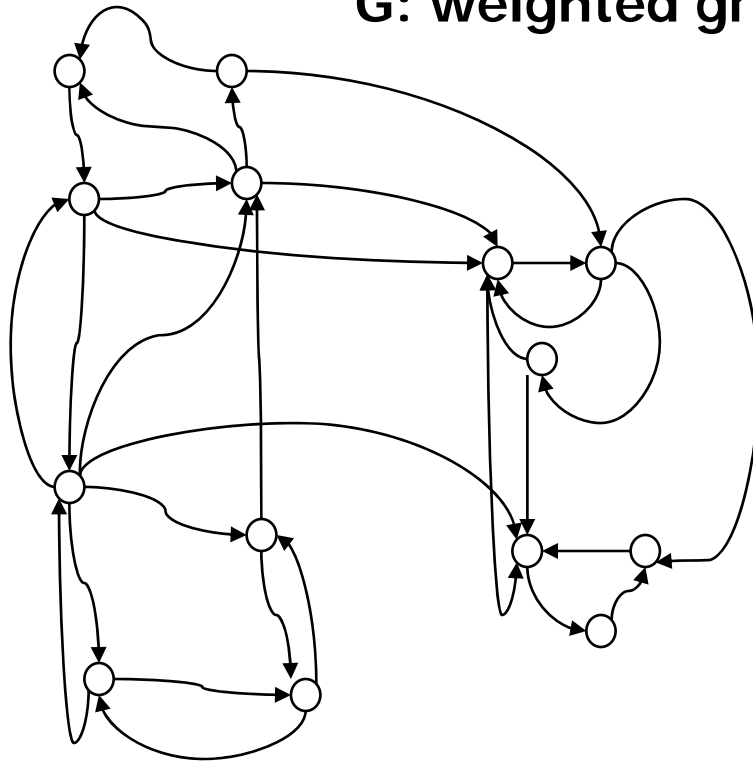


v and **w** are both redundant
 Removal of one depends on presence of other.

Observation: If no zero- or negative cycles then SAFE to remove all redundancies.

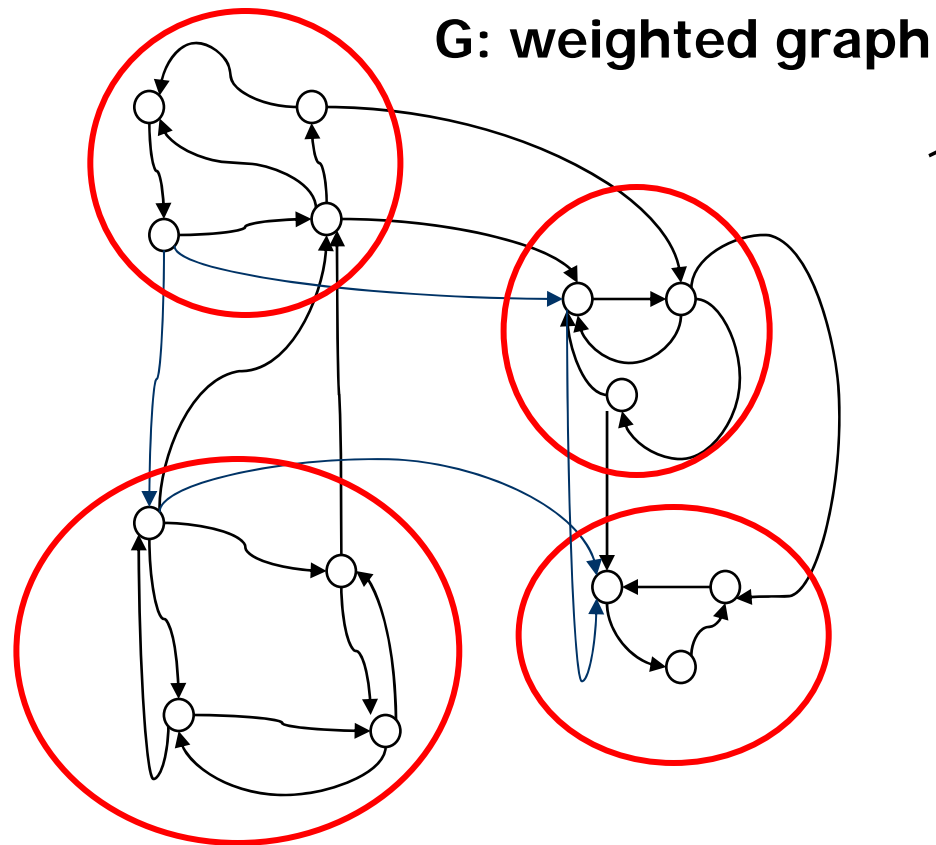
Shortest Path Reduction Solution

G: weighted graph



Shortest Path Reduction

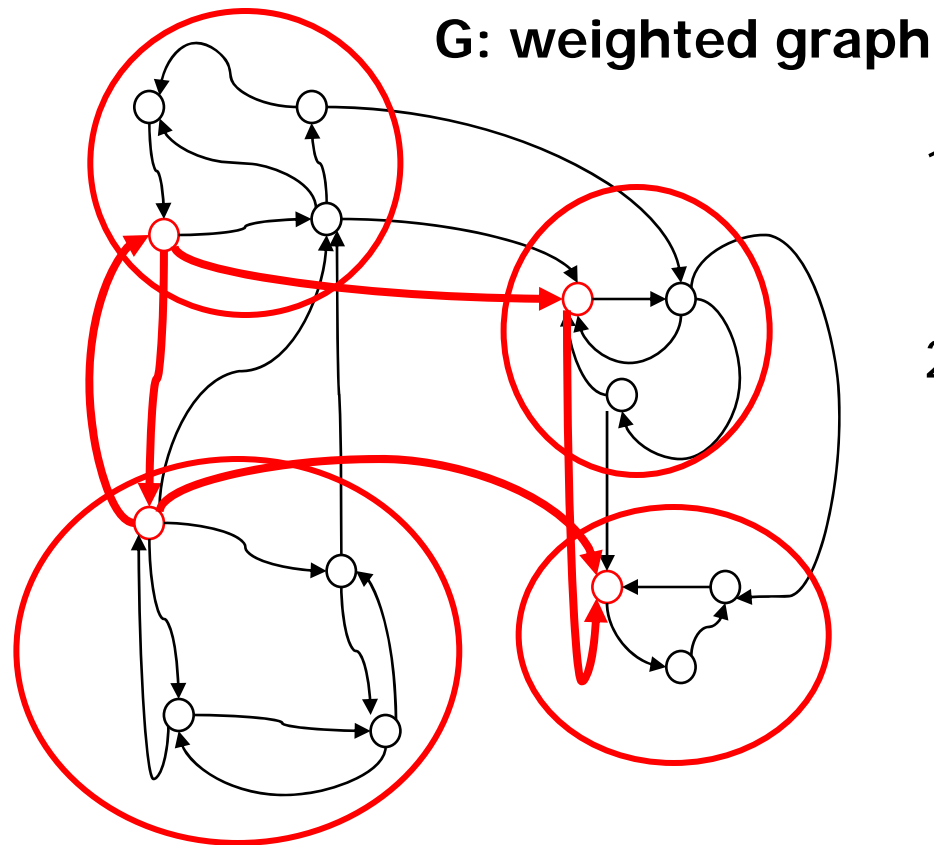
Solution



1. Equivalence classes based on 0-cycles.

Shortest Path Reduction

Solution

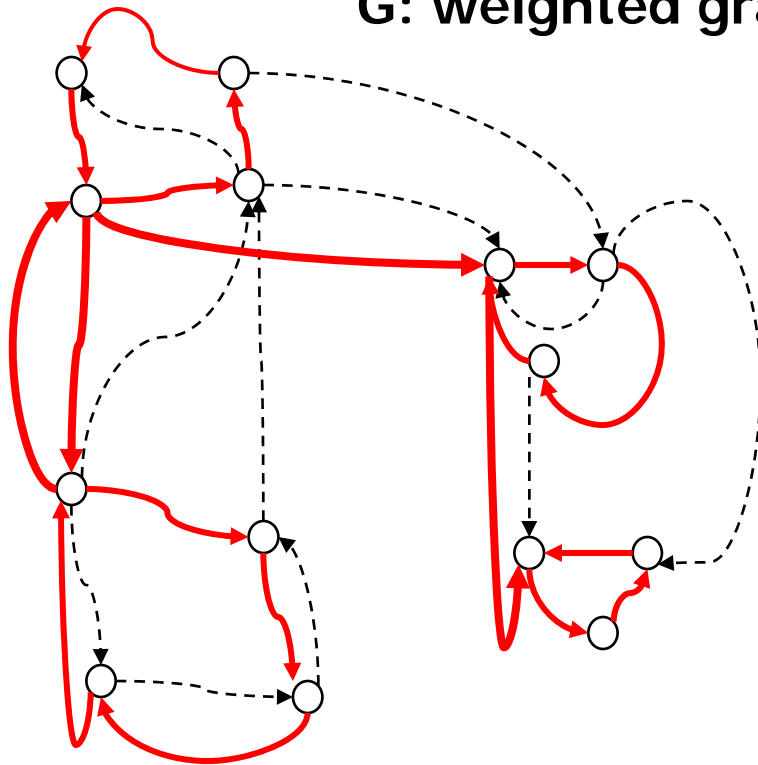


1. Equivalence classes based on 0-cycles.
2. Graph based on **representatives**.
Safe to remove redundant edges

Shortest Path Reduction

Solution

G: weighted graph

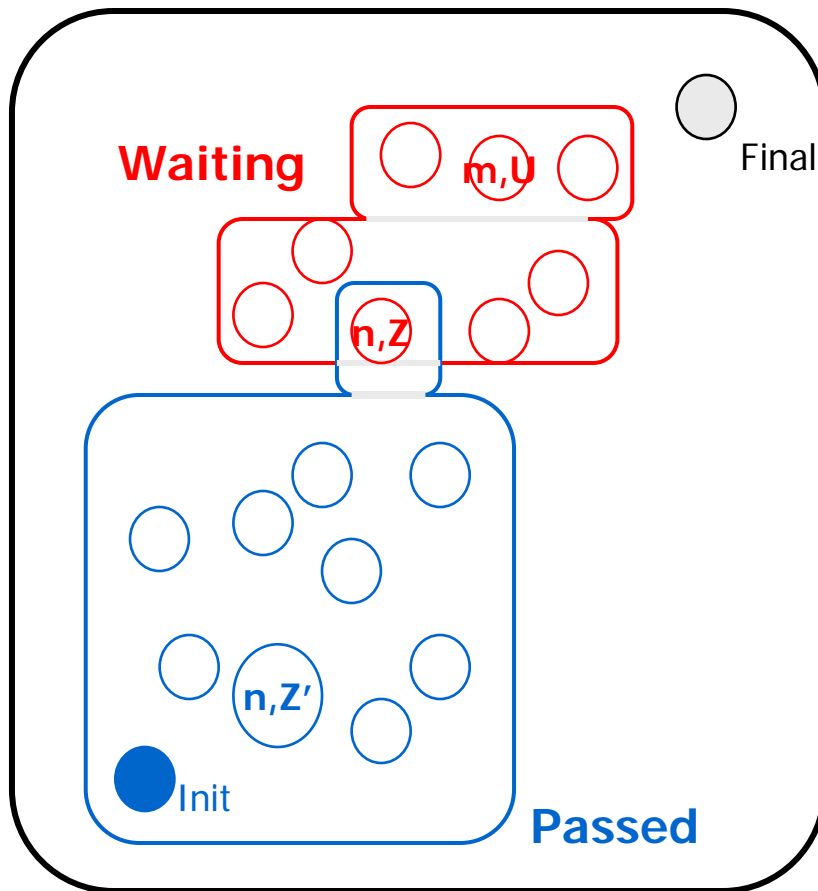


1. Equivalence classes based on 0-cycles.
2. Graph based on **representatives**.
Safe to remove redundant edges
3. **Shortest Path Reduction**
 =
 One cycle pr. class
 +
 Removal of redundant edges
 between classes

Canonical given order of clocks

Earlier Termination

Init -> Final ?



INITIAL **Passed** := \emptyset ;
Waiting := $\{(n_0, Z_0)\}$

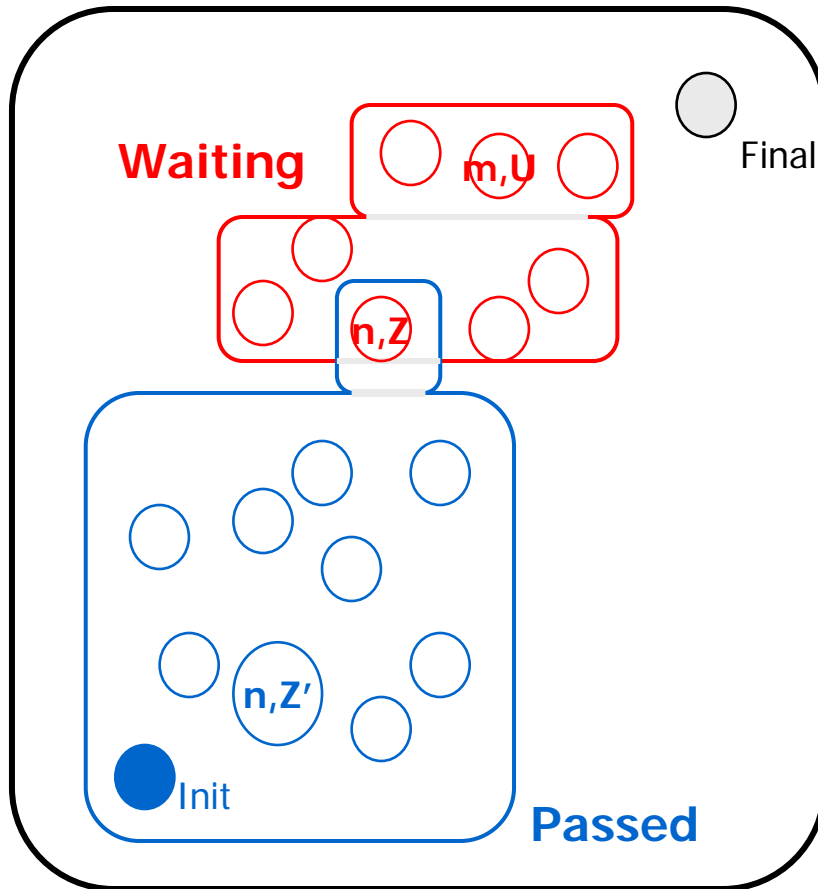
REPEAT

- pick (n, Z) in **Waiting**
- **if** for some $Z' \supseteq Z$
 (n, Z') in **Passed** then **STOP**
- **else** /explore/ add
 $\{ (m, U) : (n, Z) \Rightarrow (m, U) \}$
to **Waiting**;
Add (n, Z) to **Passed**

UNTIL **Waiting** = \emptyset
or
Final is in **Waiting**

Earlier Termination

Init -> Final ?



INITIAL **Passed** := \emptyset ;
Waiting := $\{(n_0, Z_0)\}$

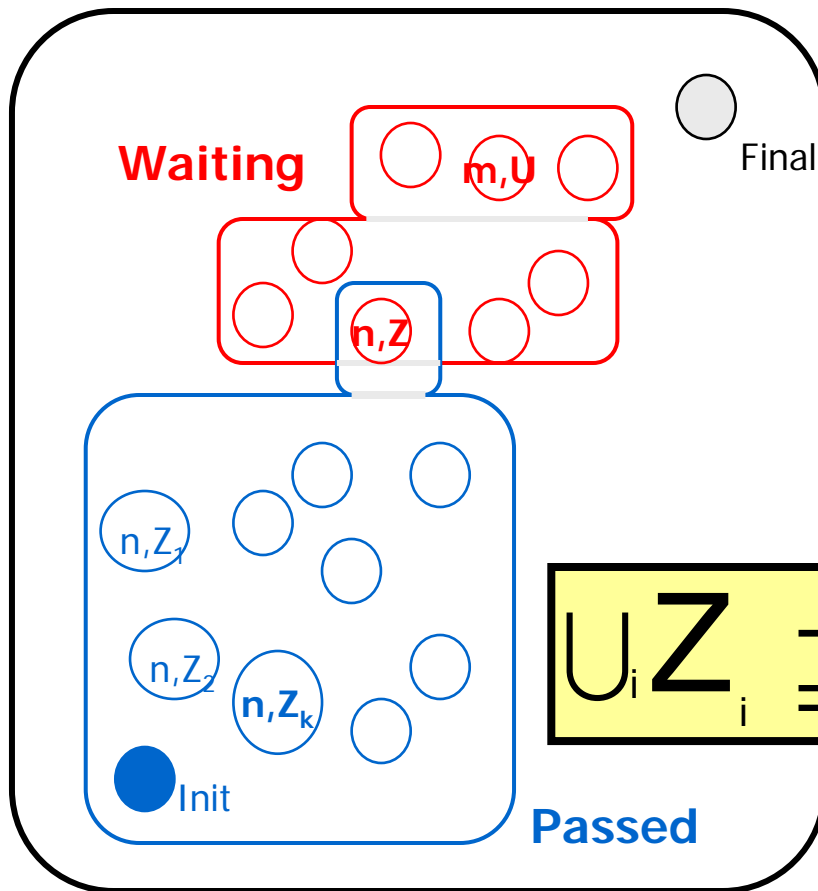
REPEAT

- pick (n, Z) in **Waiting**
- **if** for some $Z' \supseteq Z$
 (n, Z') in **Passed** then **STOP**
- **else** /explore/ add
 $\{(m, U) : (n, Z) \Rightarrow (m, U)\}$
to **Waiting**;
Add (n, Z) to **Passed**

UNTIL **Waiting** = \emptyset
or
Final is in **Waiting**

Earlier Termination

Init -> Final ?



```
INITIAL Passed := ∅;
      Waiting := {(n0,Z0)}
```

REPEAT

- pick (n,Z) in **Waiting**
- if for some $Z' \supseteq Z$
- $(n,Z) \in$ **Passed** then STOP
- else explore/ add
- $\{ (m,U) : (n,Z) \Rightarrow (m,U) \}$
- to **Waiting**;
- Add (n,Z) to **Passed**

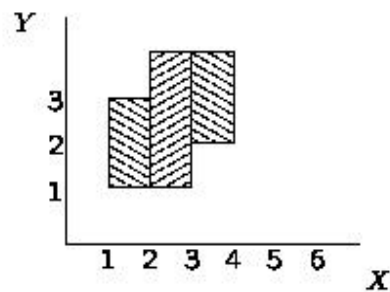
```
UNTIL Waiting = ∅
      or
      Final is in Waiting
```

Clock Difference Diagrams

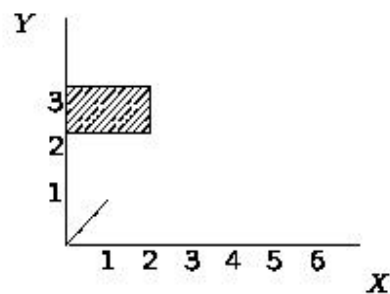
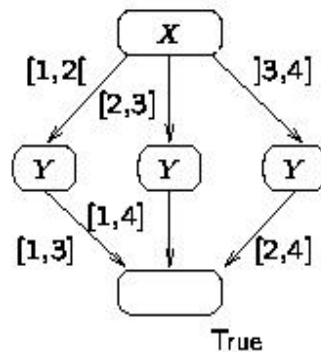
= Binary Decision Diagrams + Difference Bounded Matrices

CAV99

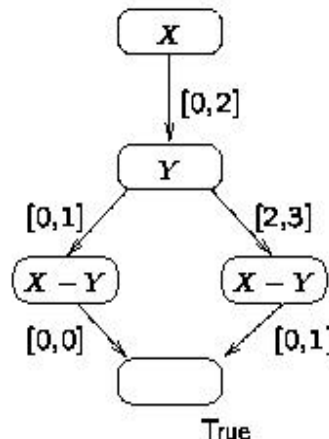
CDD-representations



(b)



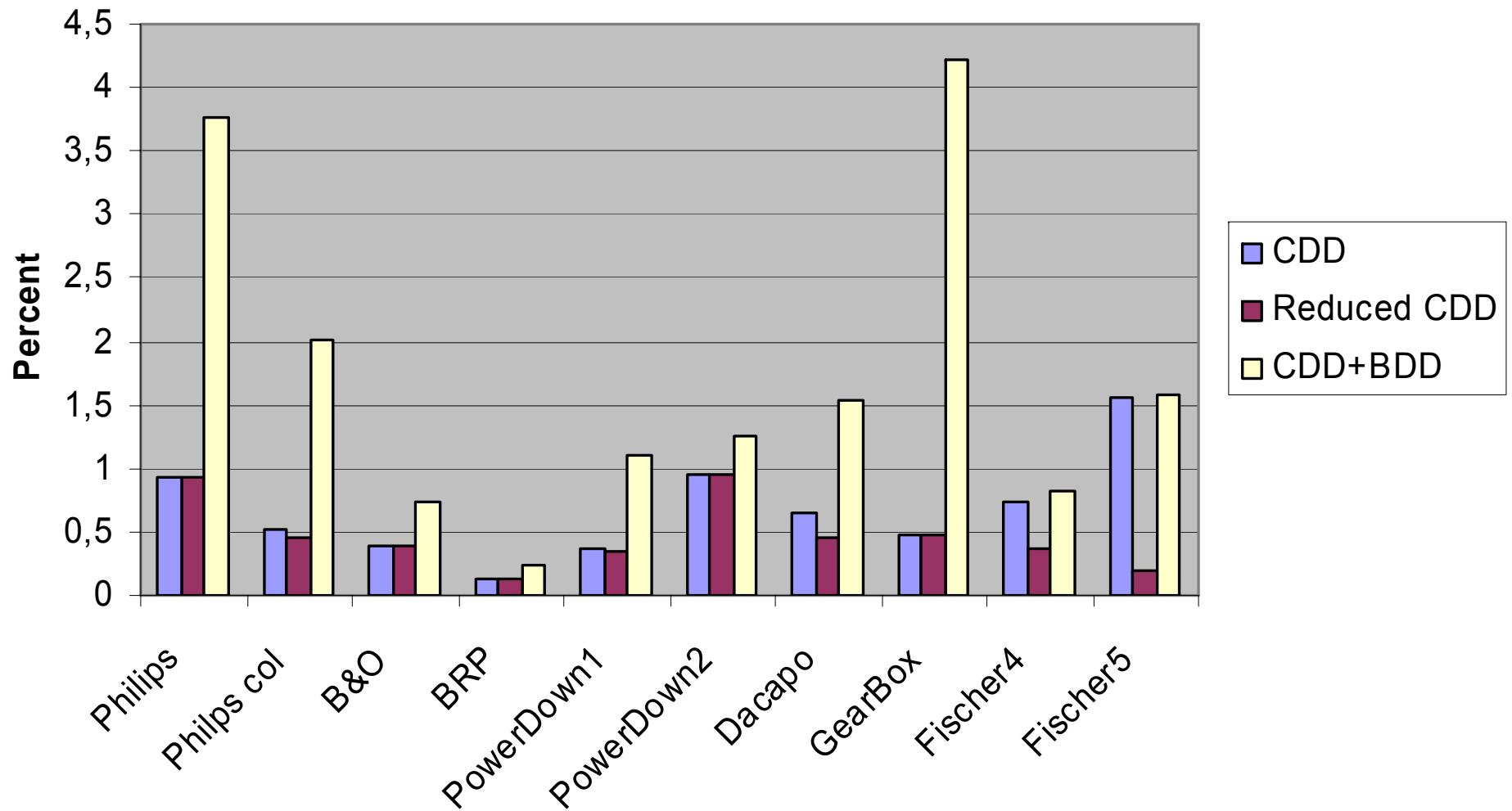
(c)



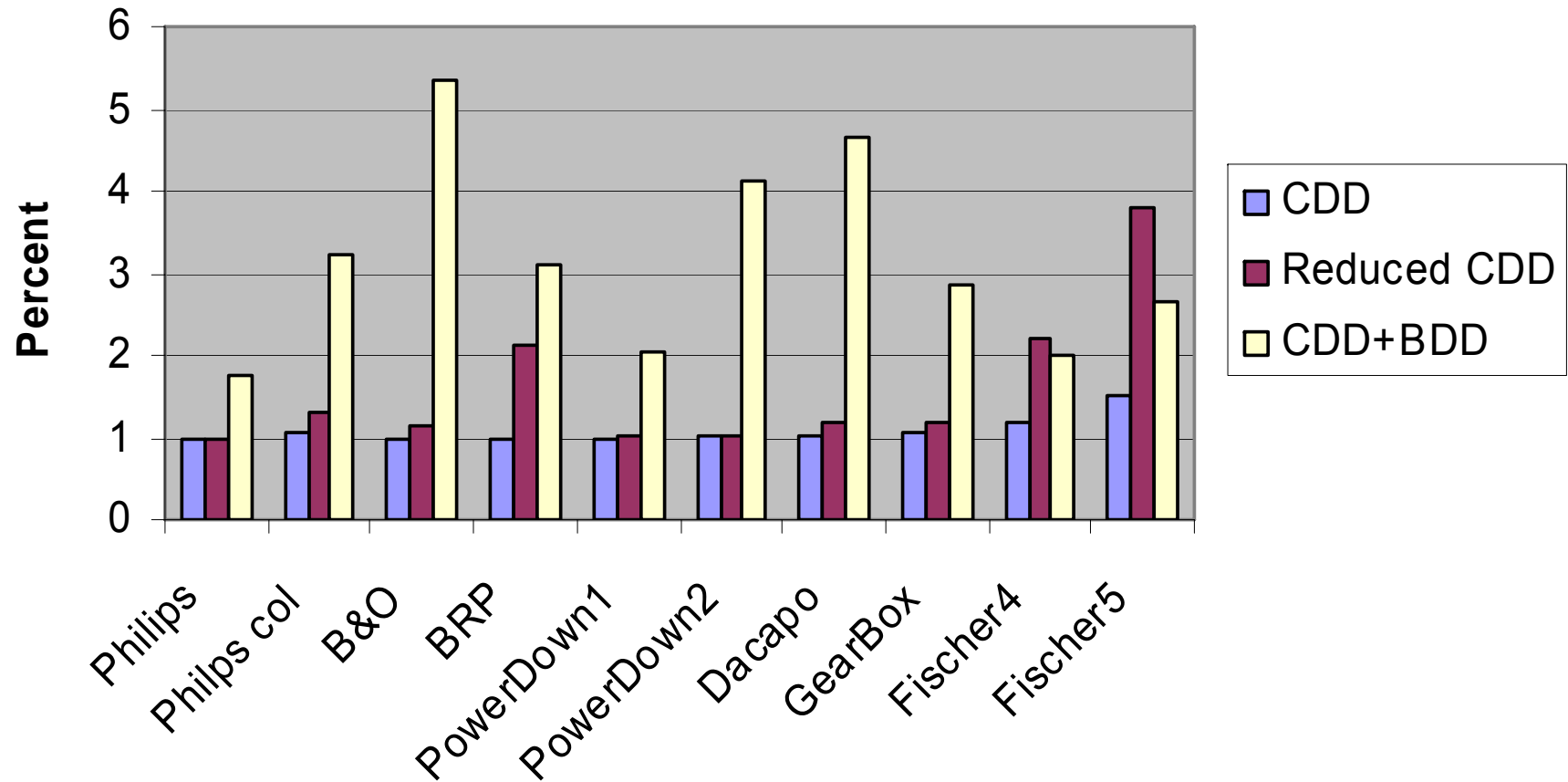
- Nodes labeled with differences
- Maximal sharing of substructures (also across different CDDs)
- Maximal intervals
- Linear-time algorithms for set-theoretic operations.

- NDD's [Maler et. al](#)
- DDD's [Møller, Lichtenberg](#)

SPACE PERFORMANCE

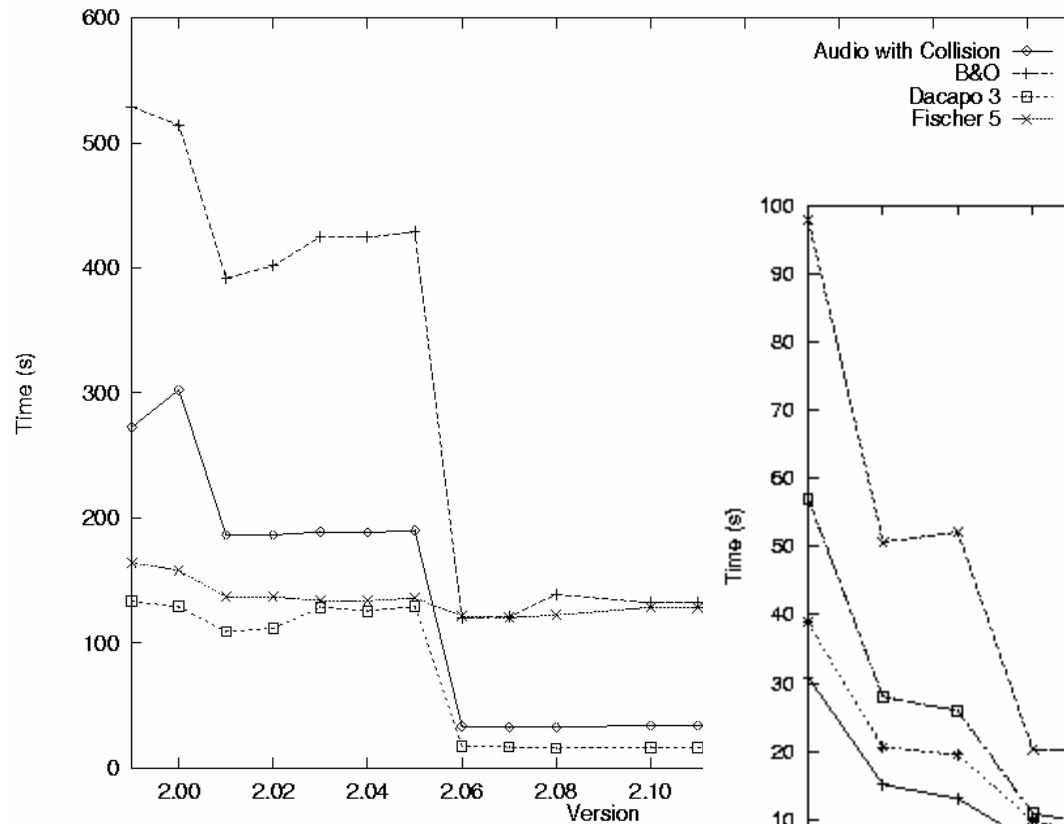


TIME PERFORMANCE

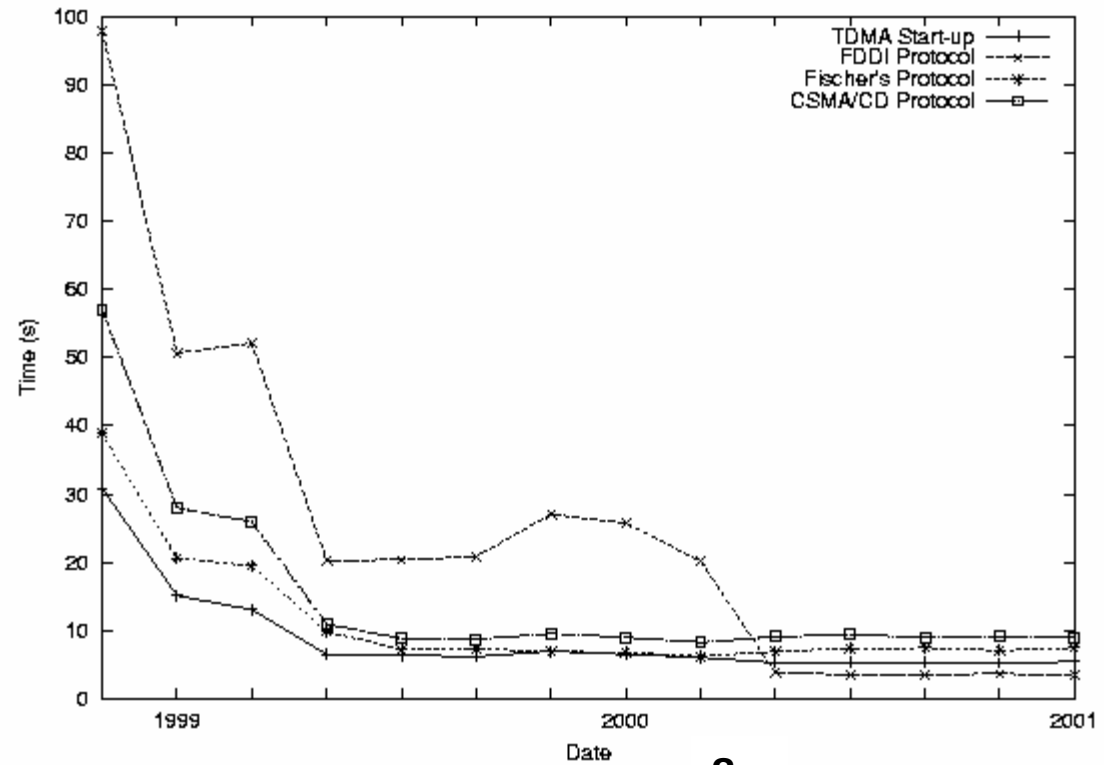


UPPAAL 1995 - 2001

Every 9 month
 10 times better
 performance!



Dec'96



3.x

Liveness Checking



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Liveness Properties

in UPPAAL

$F ::= E \square P$ | ————— **Possibly always P**

$A \diamond P$ | ————— **Eventually P**
 is equivalent to $(\neg E \square \neg P)$

$P \rightarrow Q$ | ————— **P leads to Q**
 is equivalent to $A \square (P \Rightarrow A \diamond Q)$

Bouajjani, Tripakis, Yovine'97
On-the-fly symbolic model checking of TCTL

Liveness

$E[]\phi$ $(A\Diamond\neg\phi)$

```

proc Liveness( $s_0, \varphi, Passed$ )  $\equiv$ 
  pre( $s_0 = delay(s_0)$ )
  pre( $s_0 \models \varphi$ )
  pre( $\neg unboundeds_0 \wedge \neg deadlocked(s_0)$ )
  pre( $\forall s \in Passed. s \models A\Diamond\neg\varphi$ )
   $WS := \{s_0\};$ 
   $ST := \emptyset;$ 
  while  $WS \neq \emptyset$  do
     $s := pop(WS);$ 
    while  $top(ST) \neq parent(s)$  do
       $Passed := Passed \cup \{pop(ST)\};$ 
    od
    push( $ST, s$ );
    if  $\forall s' \in Passed. s \not\subseteq s'$ 
      then foreach  $t : s \xrightarrow{a} t$  do
        if  $t \models \varphi$  then  $t := delay(t);$ 
          if  $unbounded(t)$  then exit( $true$ ) fi
          if  $deadlocked(t)$  then exit( $true$ ) fi
          if  $\exists t' \in ST. t = t'$  then exit( $true$ ) fi
           $push(WS, t);$ 
        fi
      od
    fi
  od
  exit( $false$ );
end

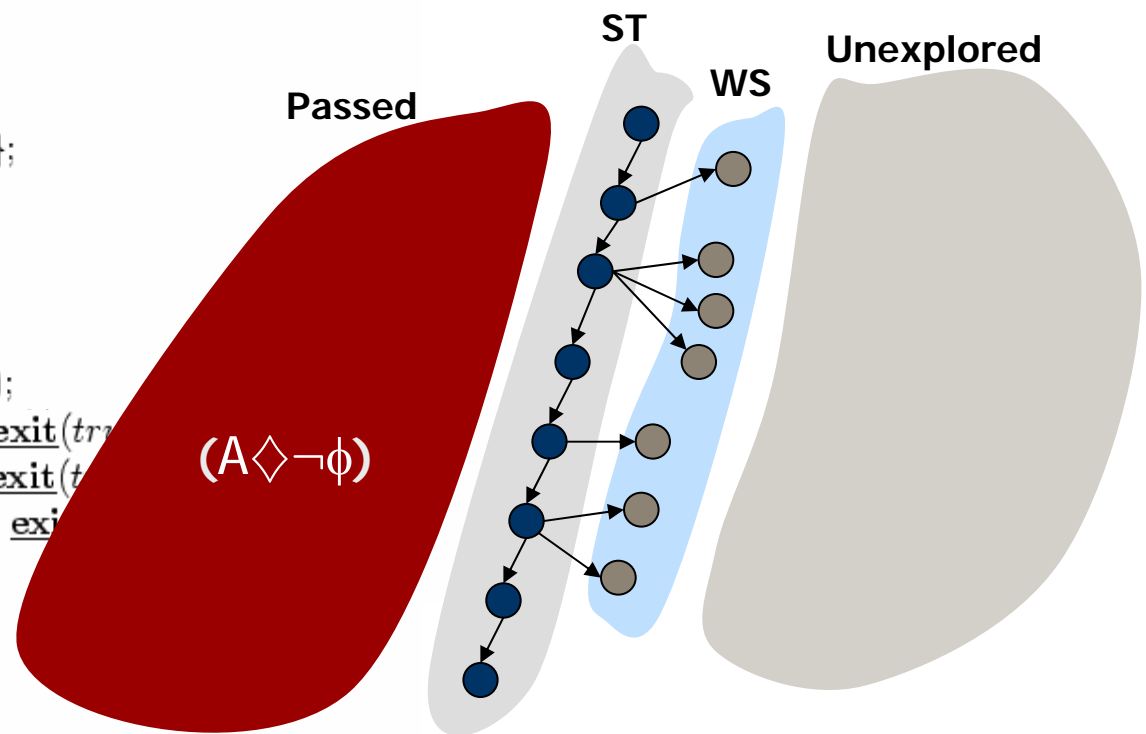
```

Liveness

$E[]\phi$ $(A\Diamond\neg\phi)$

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    od
    push(ST, s);
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      then foreach  $t : s \xrightarrow{a} t$  do
        if  $t \models \varphi$  then  $t := delay(t)$ ;
        if unbounded(t) then exit(tr);
        if deadlocked(t) then exit(tr);
        if  $\exists t' \in ST. t = t'$  then exit(tr);
        push(WS, t);
      fi
    od
  fi
  exit(false);
end
  
```

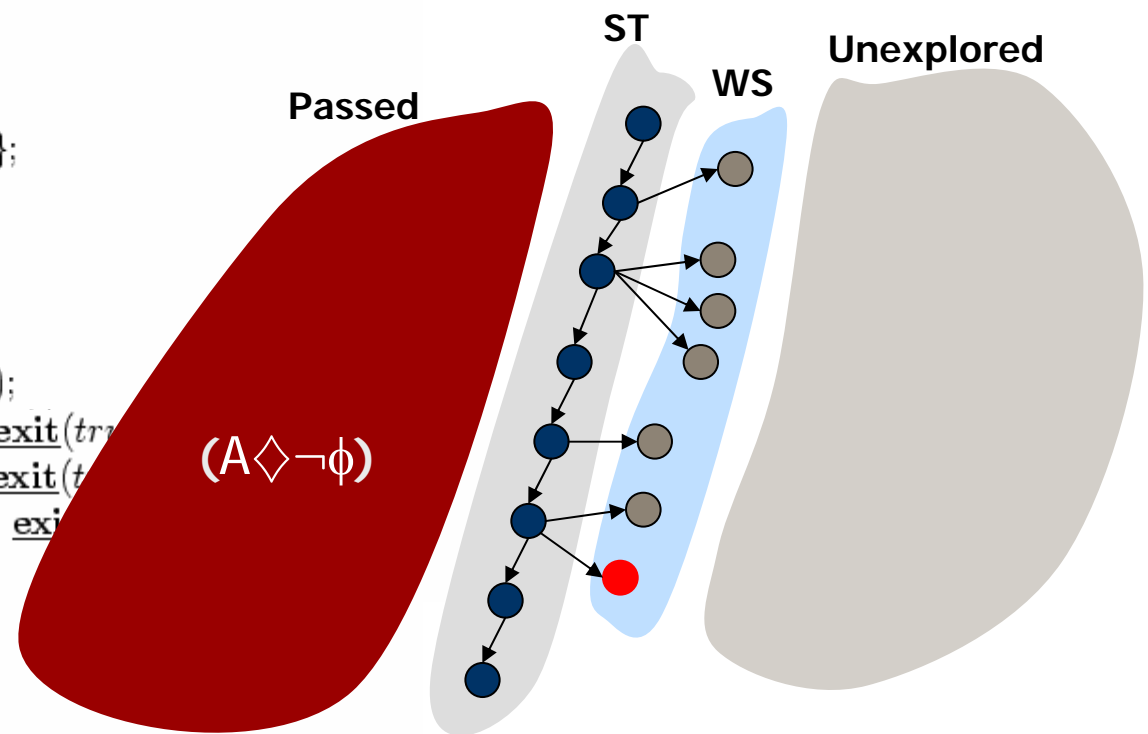


Liveness

$$E\Box\phi \quad (A\Diamond\neg\phi)$$

```

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    od
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      if  $\exists t' \in ST. t = t'$  then exit( $t$ );
      push(WS, t);
    fi
  od
fi
od
exit(false);
end
  
```

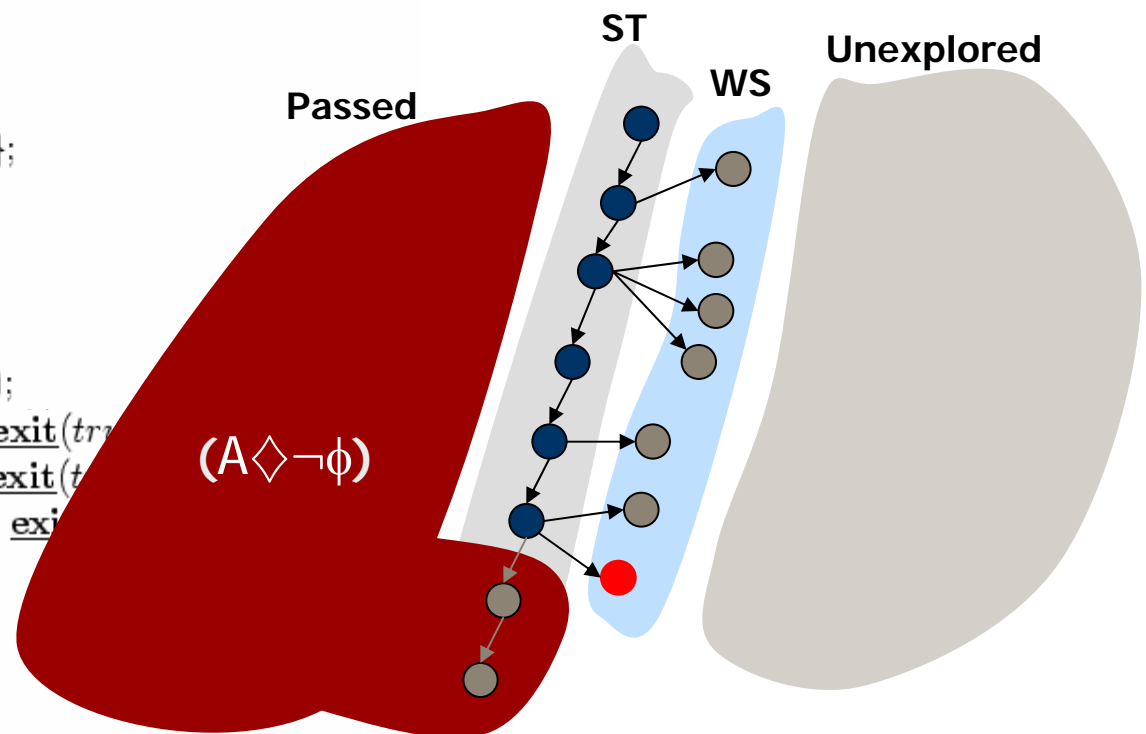


Liveness

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    od
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        if  $t \models \varphi$  then  $t := delay(t)$ ;
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        if  $\exists t' \in ST. t = t'$  then exit( $t$ );
        push(WS,  $t$ );
      fi
    od
  fi
  exit(false);
end
  
```



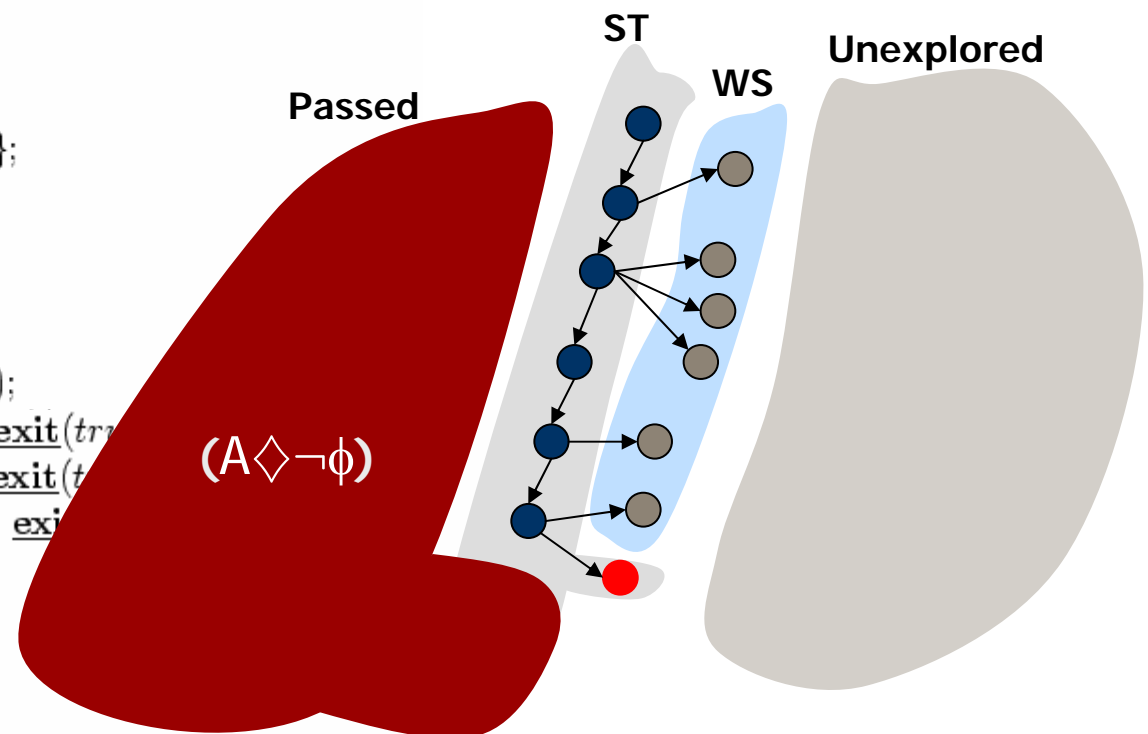
Liveness

$$E\Box\phi \quad (A\Diamond\neg\phi)$$

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        push( $WS, t$ );
      fi
    od
  fi
  exit( $false$ );
end

```

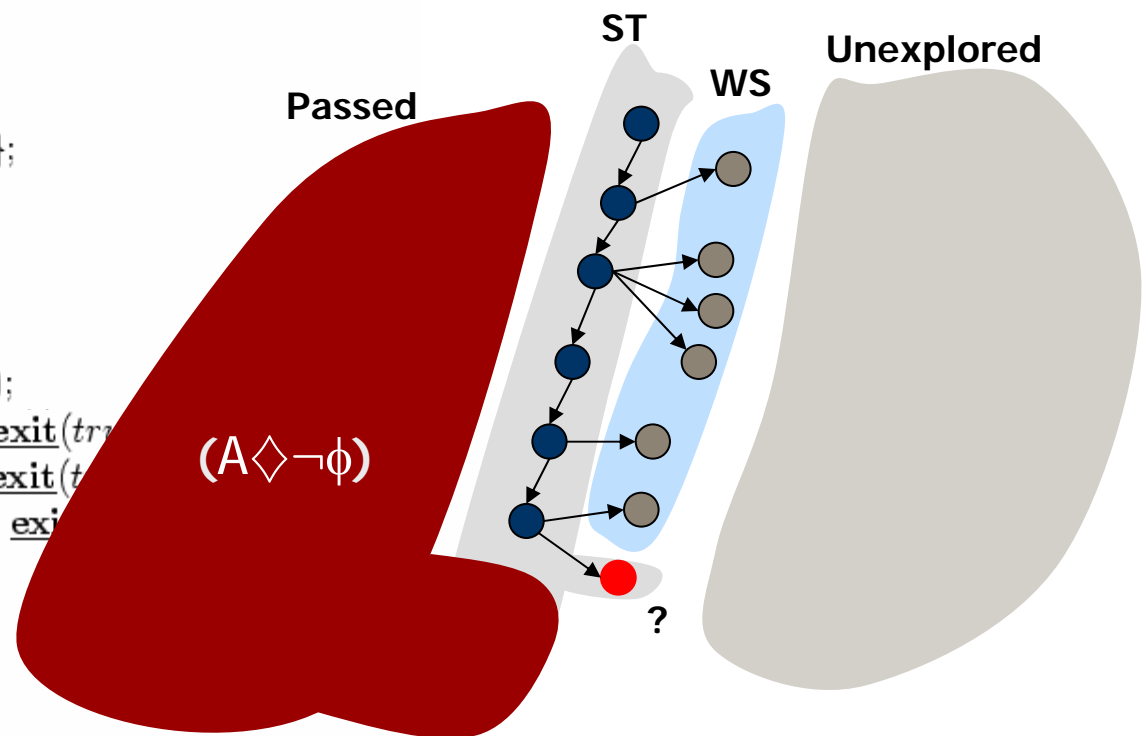


Liveness

$$E\Box\phi \quad (A\Diamond\neg\phi)$$

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        push(WS, t);
      od
    fi
  od
  exit(false);
end
  
```

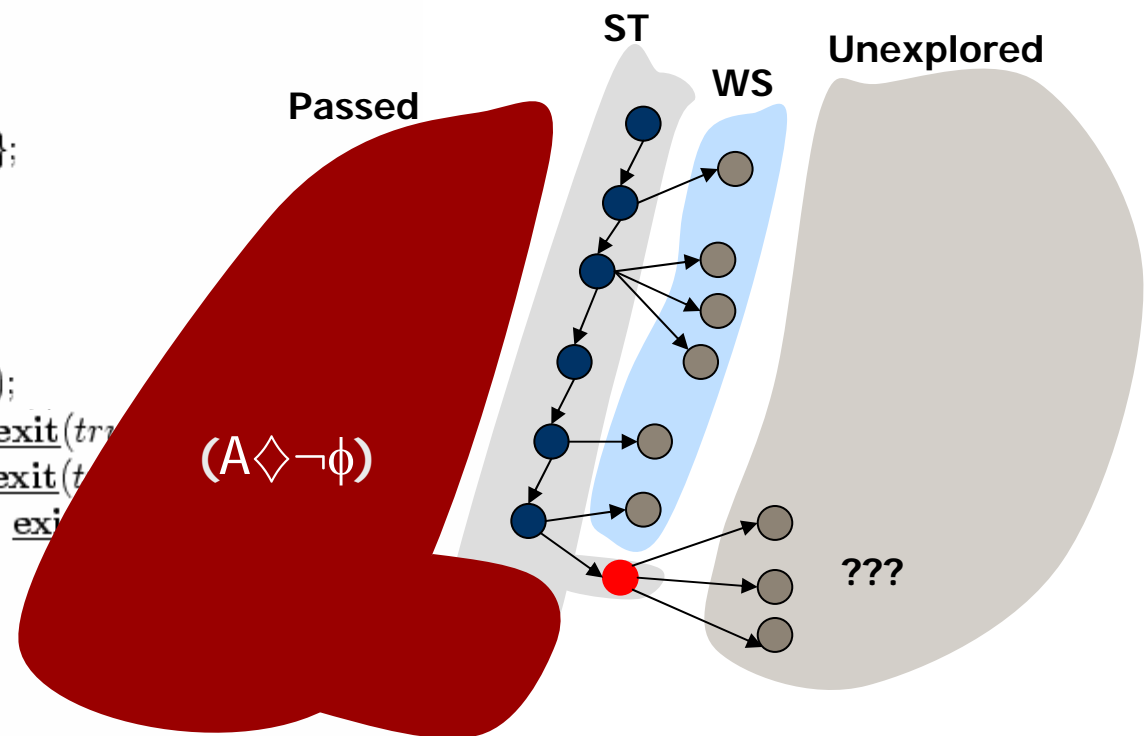


Liveness

$$E\Box\phi \quad (A\Diamond\neg\phi)$$

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      if  $unbounded(t)$  then exit( $tr$ );
      if  $deadlocked(t)$  then exit( $t$ );
      if  $\exists t' \in ST. t = t'$  then exit( $t$ );
      push( $WS, t$ );
    fi
  od
  fi
  od
  exit( $false$ );
end
  
```



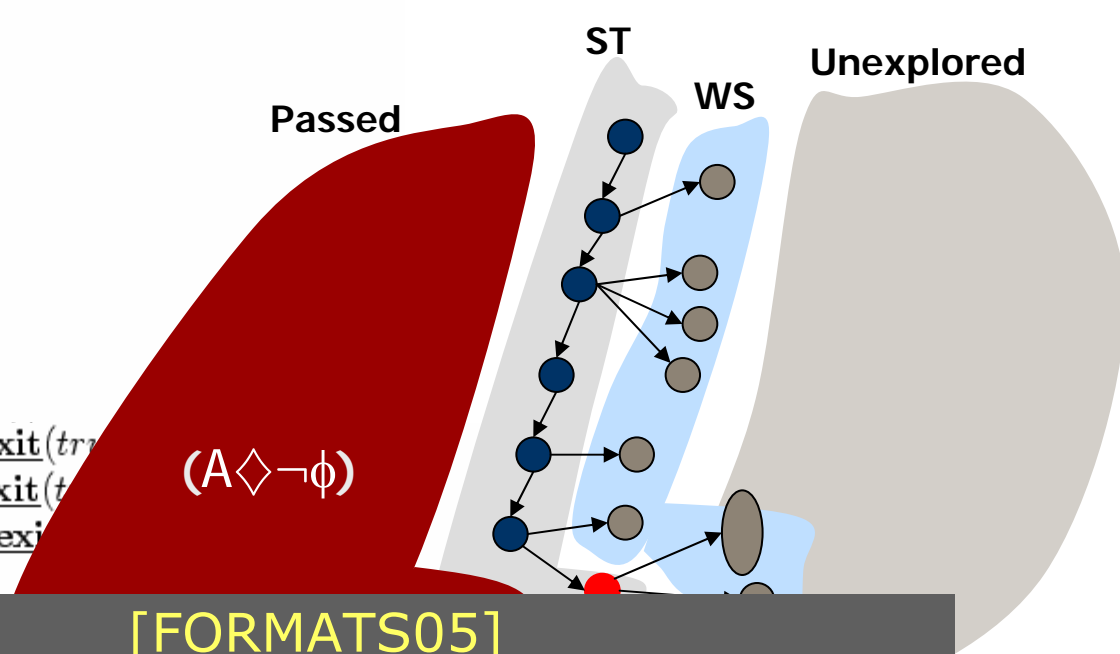
Liveness

$$E\Box\phi \quad (A\Diamond\neg\phi)$$

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      if deadlocked( $t$ ) then exit( $t$ );
      if  $\exists t' \in ST. t = t'$  then exit( $t$ );
      push(WS, t);
    fi
  od
  exit(false);
end

```



[FORMATS05]
Extensions allowing for automatic synthesis of smallest bound t such that $A\Diamond_{\leq t}\phi$ holds

Verification Options



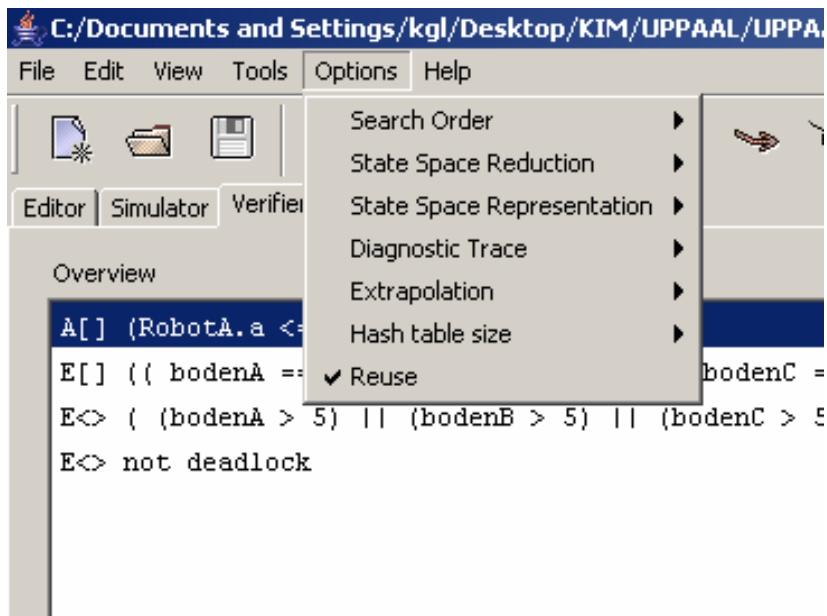
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Verification Options



Search Order

- Depth First
- Breadth First

State Space Reduction

- None
- Conservative
- Aggressive

State Space Representation

- DBM
- Compact Form
- Under Approximation
- Over Approximation

Diagnostic Trace

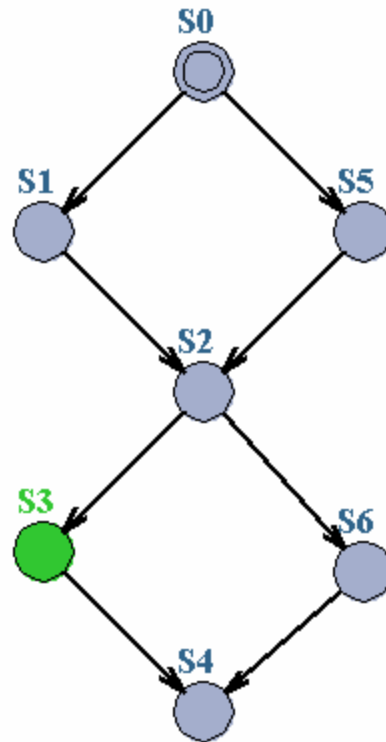
- Some
- Shortest
- Fastest

Extrapolation

Hash Table size

Reuse

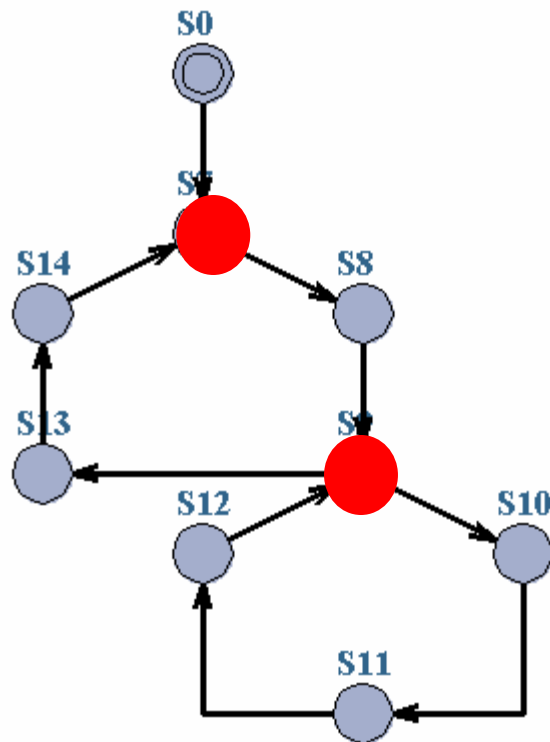
State Space Reduction



However,
Passed list useful for
 efficiency

No Cycles: **Passed** list not needed for *termination*

State Space Reduction



Cycles:

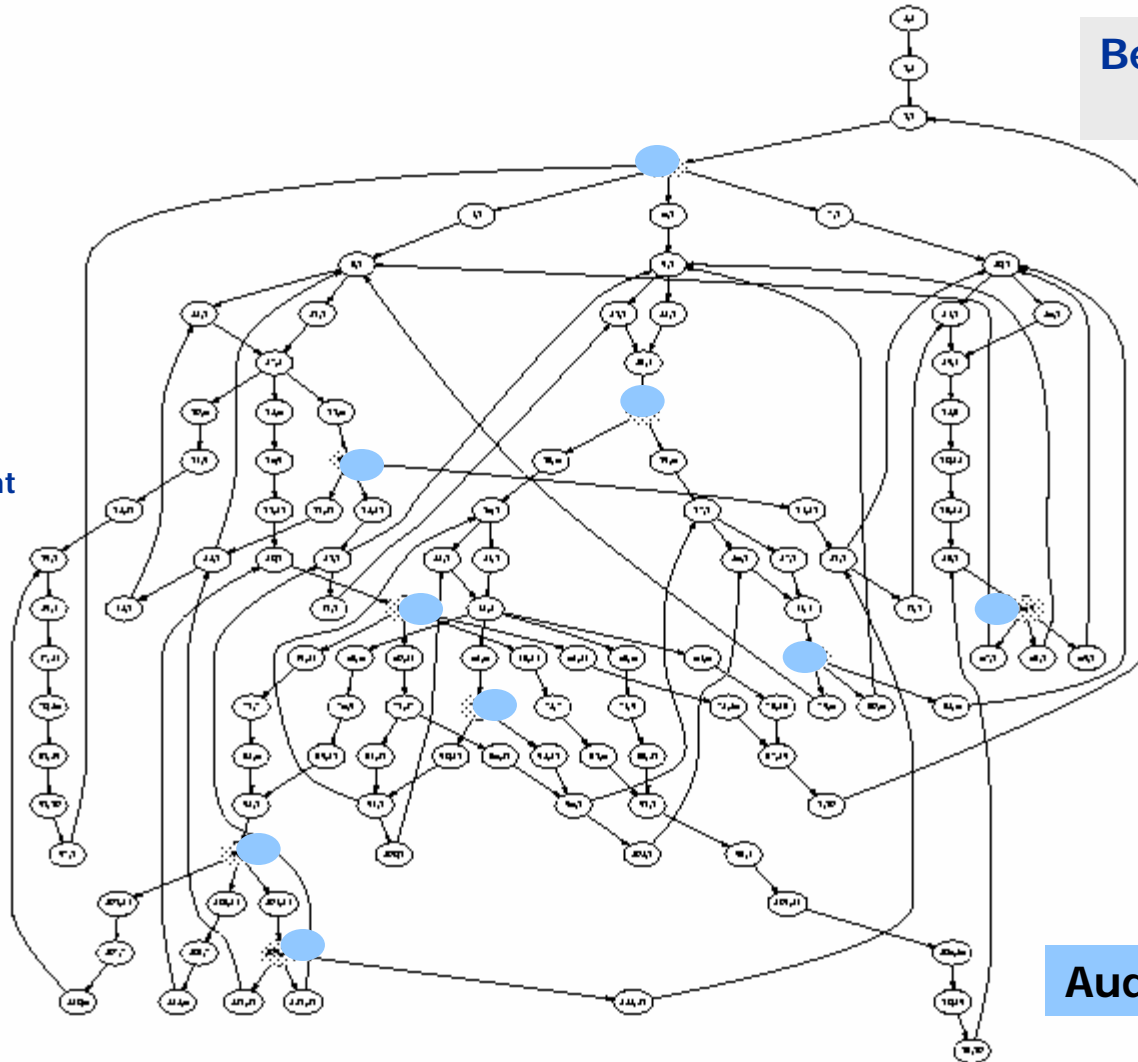
Only symbolic states involving loop-entry points need to be saved on **Passed** list

To Store or Not To Store

Behrmann, Larsen,
Pelanek 2003

117 states_{total}
→
81 states_{entrypoint}
→
9 states

Time OH
less than 10%



Audio Protocol

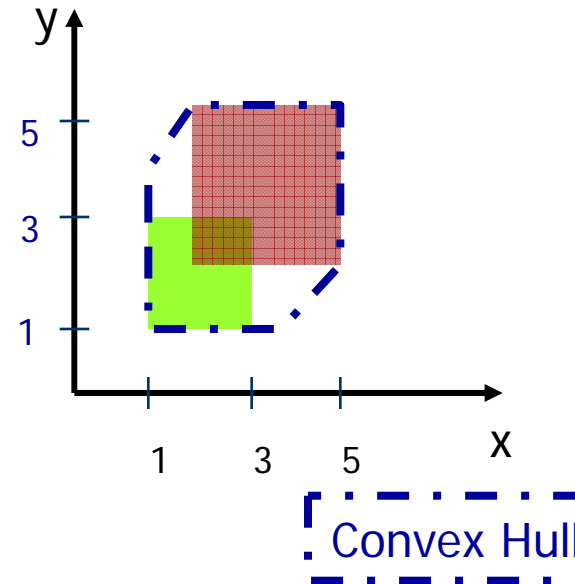
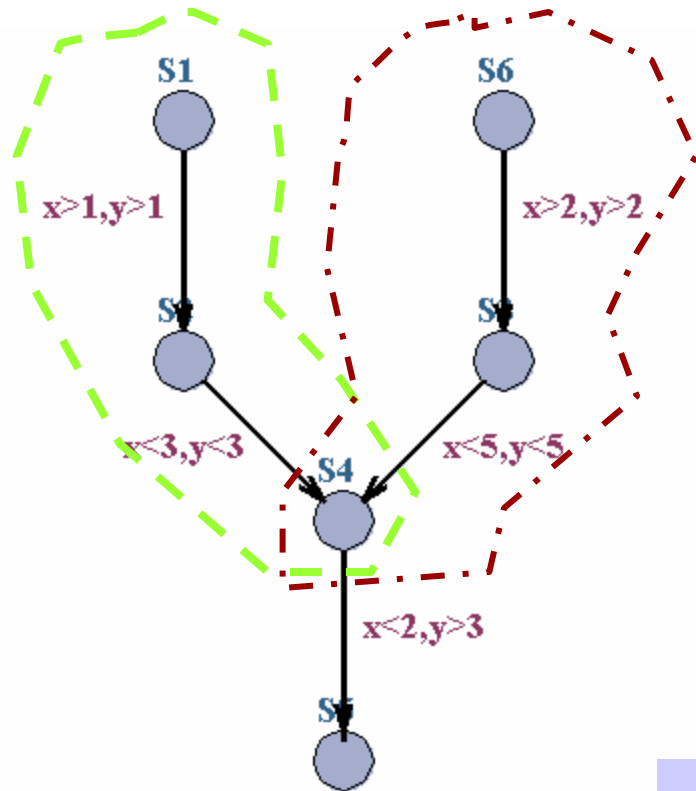
To Store or Not to Store

Behrmann, Larsen,
Pelanek 2003

	entry points	covering set	successors	random $p = 0.1$	distance $k = 10$	combination $k = 3$
Fischer 3,077	27.1% 1.00	42.1% 1.66	47.9% 1.00	53.7% 4.51	67.6% 2.76	56.9% 6.57
BRP 6,060	70.5% 1.01	16.5% 1.20	19.8% 1.03	18.3% 1.78	15.8% 1.34	7.6% 1.68
Token Ring 15,103	33.0% 1.16	10.3% 1.46	20.7% 1.03	17.2% 1.63	17.5% 1.43	16.8% 7.40
Train-gate 16,666	71.1% 1.22	27.4% 1.55	24.2% 1.68	31.8% 2.90	24.2% 2.11	19.8% 5.08
Dacapo 30,502	29.4% 1.07	24.3% 1.08	24.9% 1.07	12.2% 1.21	12.7% 1.16	7.0% 1.26
CSMA 47,857	94.0% 1.06	75.9% 2.62	81.2% 1.40	105.9% 7.66	114.9% 2.83	120.3% 6.82
BOCDP 203,557	25.2% 1.00	22.5% 1.01	6.5% 1.08	10.2% 1.02	9.3% 1.01	4.5% 1.09
BOPDP 1,013,072	14.7% 2.40	13.2% 1.33	42.1% 1.02	15.2% 1.52	11% 1.14	4.3% 1.74
Buscoupler 3,595,108	53.2% 1.29	13.6% 2.48	40.5% 1.18	31.7% 3.17	24.6% 2.13	14.3% 8.73

Over-approximation

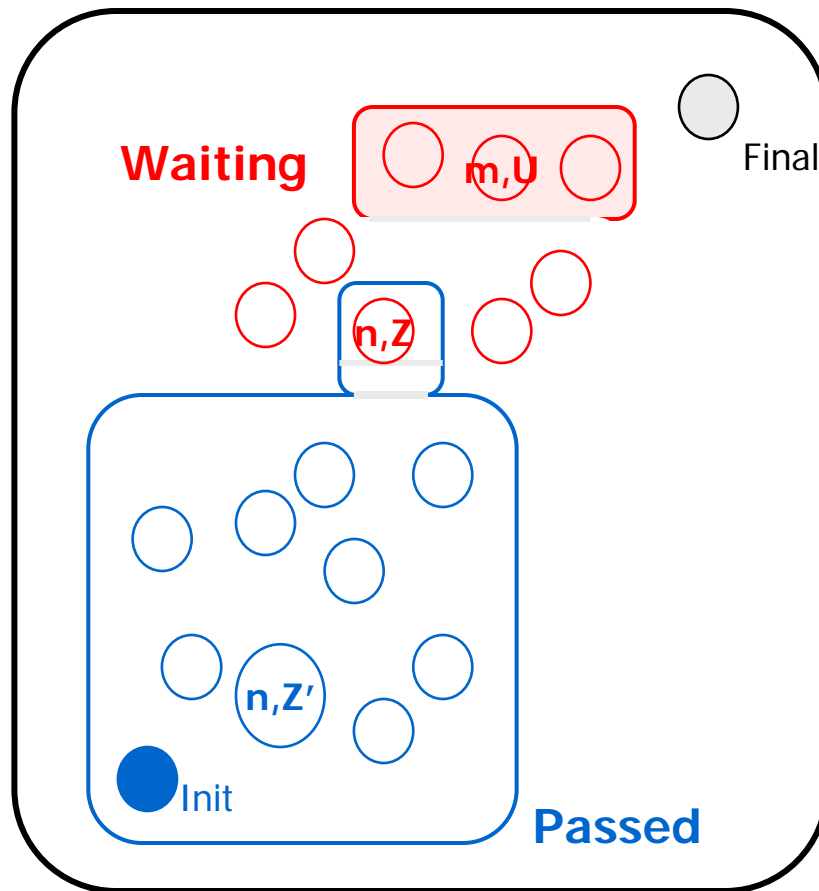
Convex Hull



TACAS04: An **EXACT** method performing as well as Convex Hull has been developed based on abstractions taking max constants into account distinguishing between clocks, locations and \leq & \geq

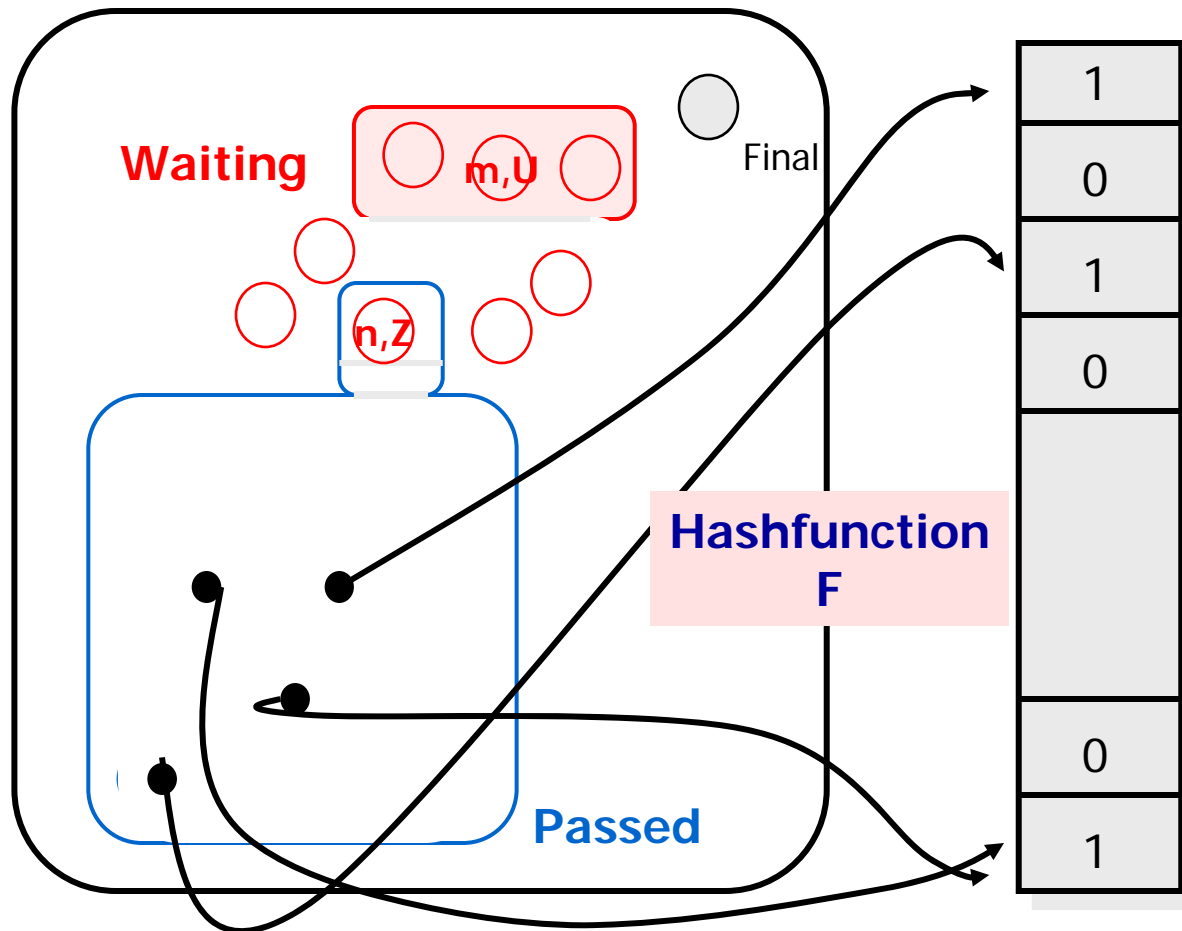
Under-approximation

Bitstate Hashing



Under-approximation

Bitstate Hashing



Passed=
Bitarray

UPPAAL
8 Mbits

Modelling Patterns



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Variable Reduction

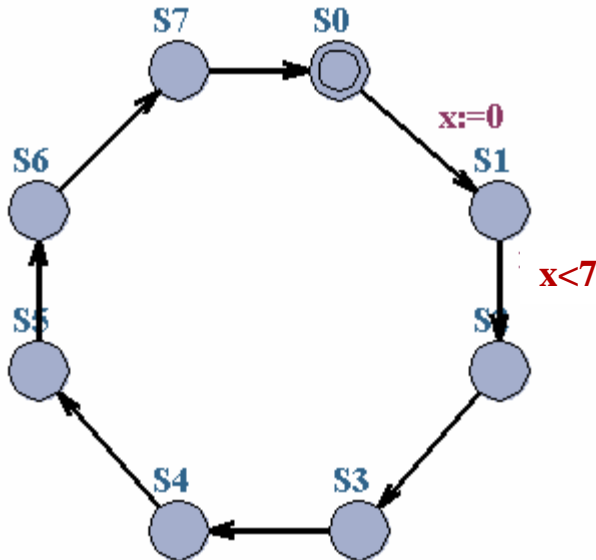
- Reduce size of state space by explicitly resetting variables when they are not used!
- Automatically performed for clock variables (active clock reduction)

```

// Remove the front element of the queue
void dequeue()
{
    int i = 0;
    len -= 1;
    while (i < len)
    {
        list[i] = list[i + 1];
        i++;
    }
    list[i] = 0;
}

```

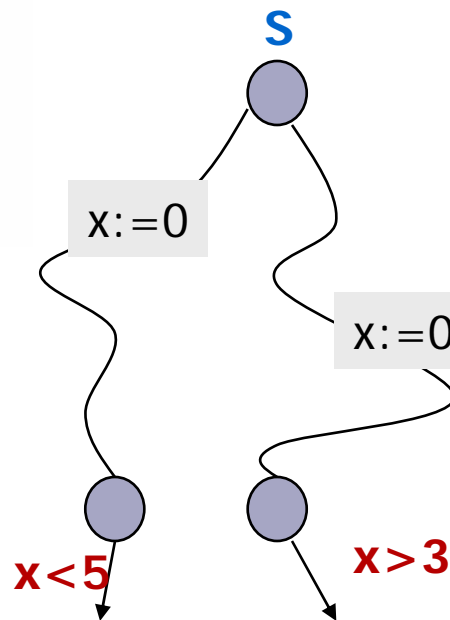
Clock Reduction



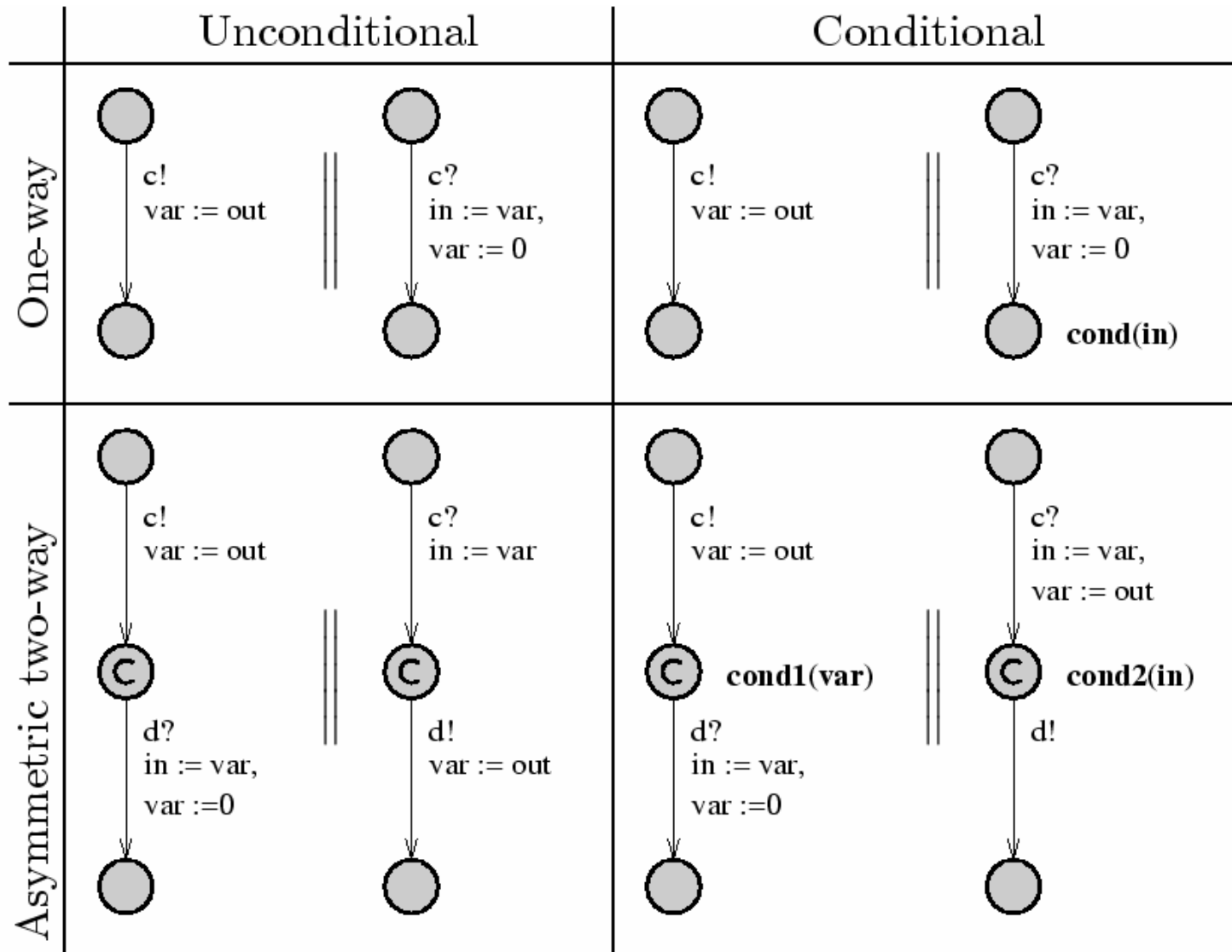
x is only *active* in location **S1**

Definition

x is *inactive* at **S** if on all path from **S**, x is always reset before being tested.

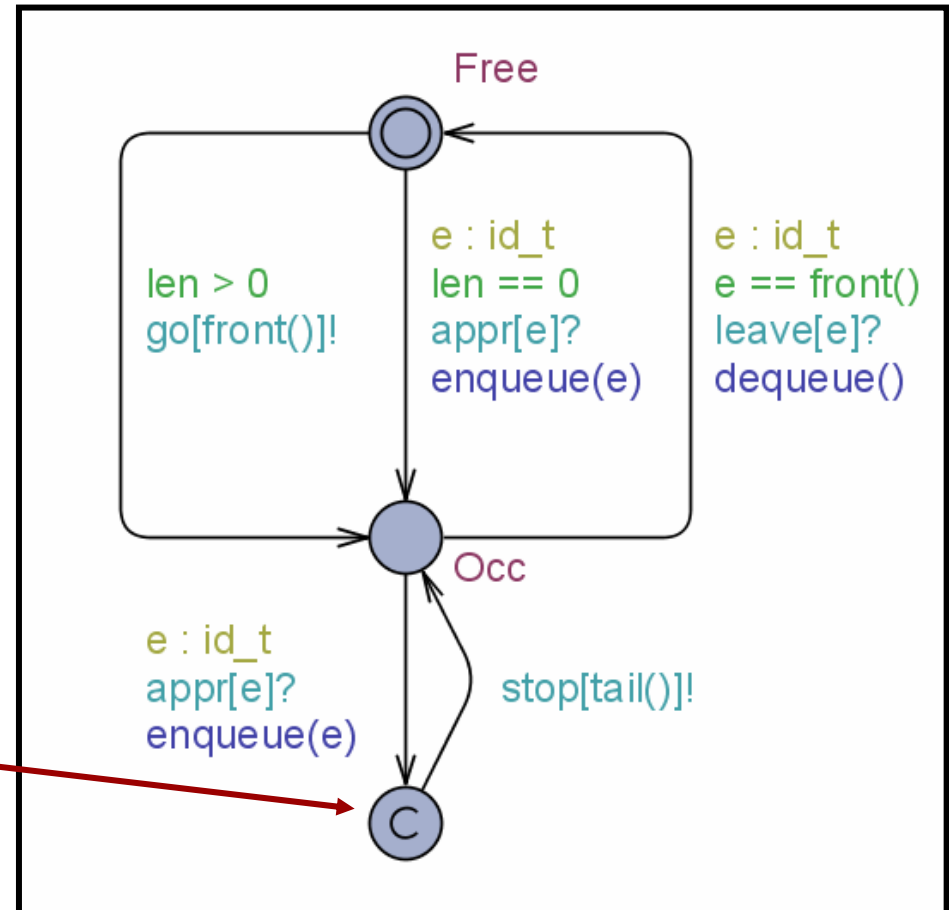


Synchronous Value Passing



Atomicity

- To allow encoding of control structure (for- or while-loops, conditionals, etc.) without erroneous interleaving
- To allow encoding of multicasting.
- Heavy use of committed locations.

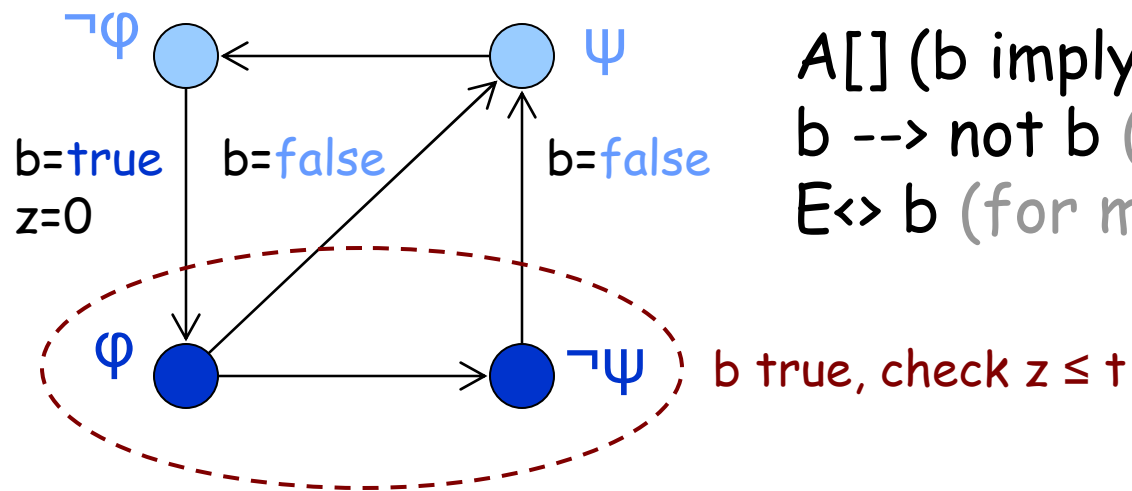


Bounded Liveness

- **Intent:** Check for properties that are guaranteed to hold eventually within some upper (time) bound.
 - Provide additional information (with a valid bound).
 - More efficient verification.
 - $\phi \text{ leadsto}_{\leq t} \psi$ reduced to $A \square (b \Rightarrow z \leq t)$ with bool b set to *true* and clock z reset when ϕ starts to hold. When ψ starts to hold, set b to *false*.

Bounded Liveness

- The truth value of b indicates whether or not ψ should hold in the future.



$A[] (b \text{ imply } z \leq t)$

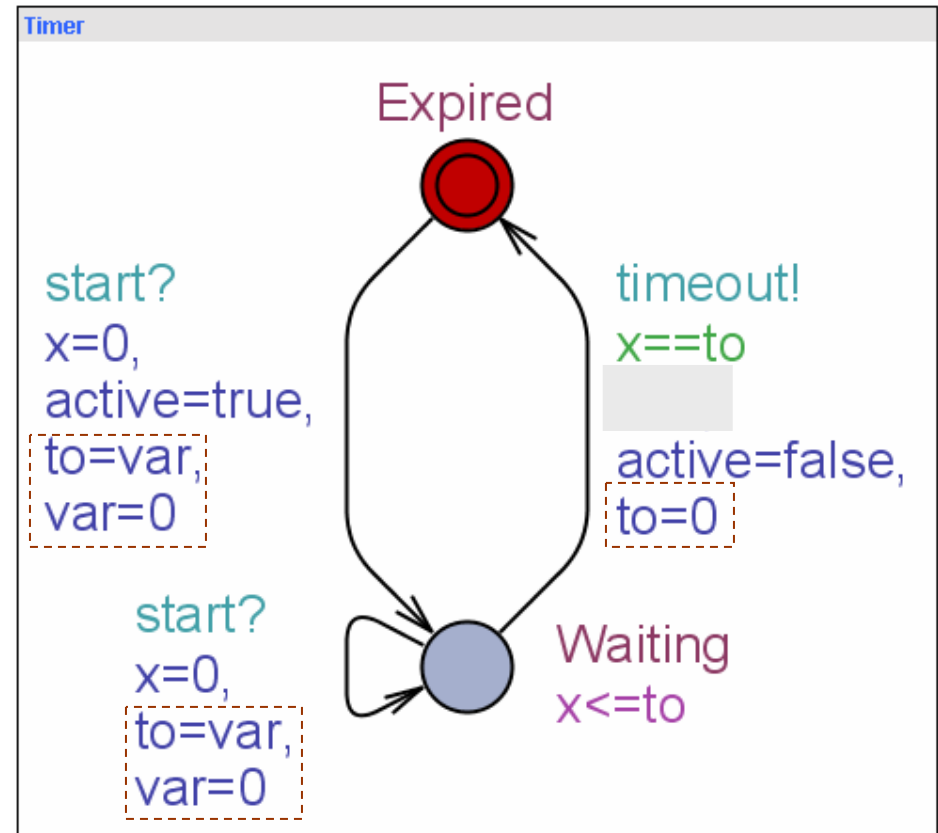
$b \dashrightarrow \text{not } b$ (for non zenoness)

$E \langle \rangle b$ (for meaningful check)

Timers

Parametric timer:

- (re-)start(value)
start! var=value
- expired?
active (bool)
active go?
(bool+urgent chan)
- time-out event
timeout?



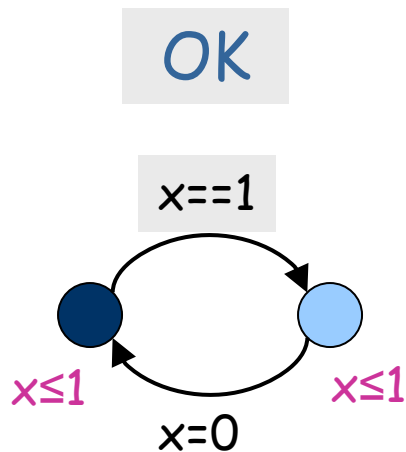
Declare 'to' with a tight range.

Zenoness

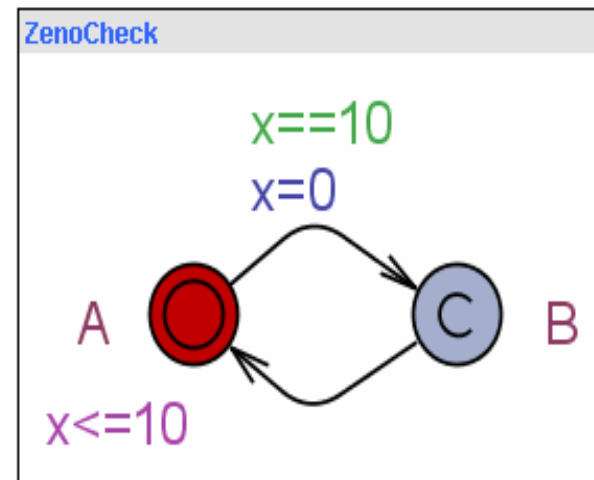
- **Problem:** UPPAAL does not check for zenoness directly.
 - A model has “zeno” behavior if it can take an infinite amount of actions in finite time.
 - That is usually not a desirable behavior in practice.
 - Zeno models may wrongly conclude that some properties hold though they logically should not.
 - Rarely taken into account.

- **Solution:** Add an observer automata and check for non-zenoness, i.e., that time will always pass.

Zenoness



Detect by
•adding the
observer:



Constant (10) can be anything (>0), but choose it well w.r.t. your model for efficiency. Clocks 'x' are local.

•and check the property

ZenoCheck.A \dashrightarrow **ZenoCheck.B**

Compositionality & Abstraction



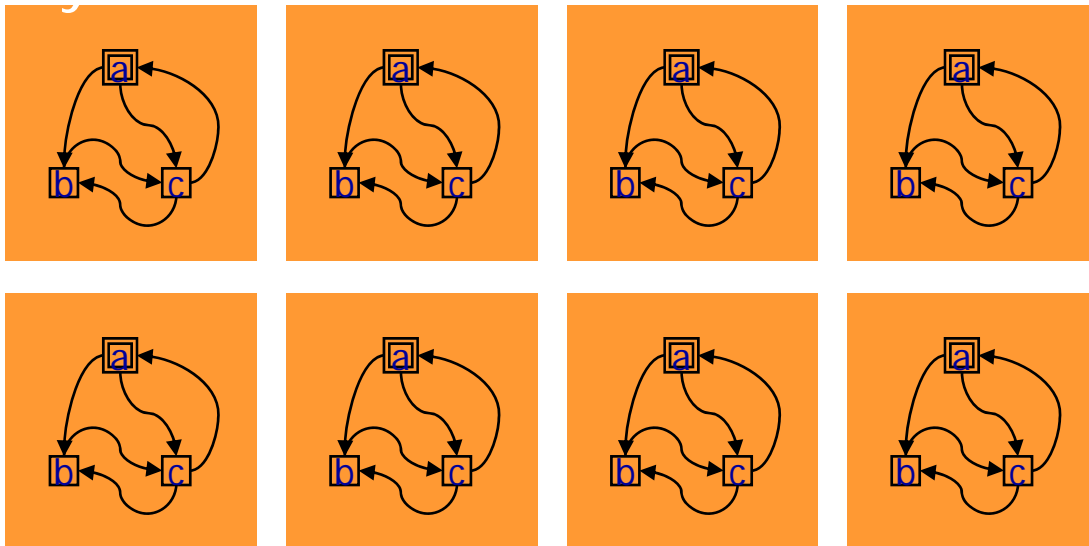
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The State Explosion Problem

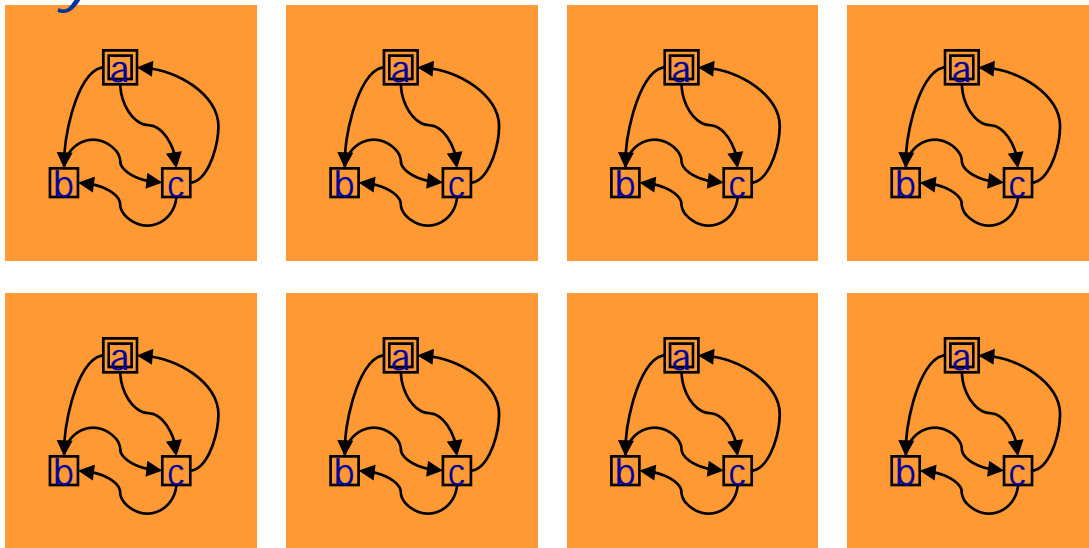


sat φ

Model-checking is either
 EXPTIME-complete or PSPACE-complete
 (for TA's this is true even for a single TA)

Abstraction

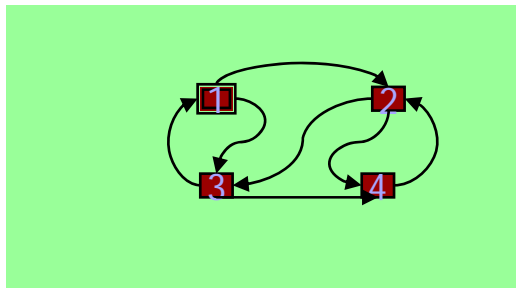
Sys



sat φ

REDUCE TO

Abs



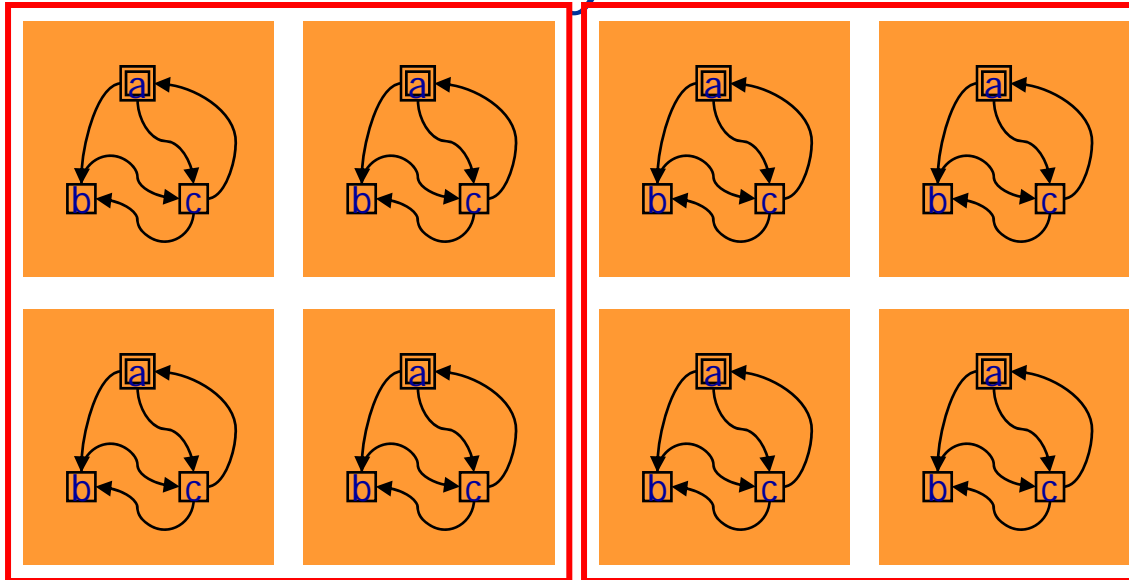
sat φ

Preserving safety properties

$\frac{Abs \text{ sat } \varphi \quad Sys \leq Abs}{Sys \text{ sat } \varphi}$
--

Compositionality

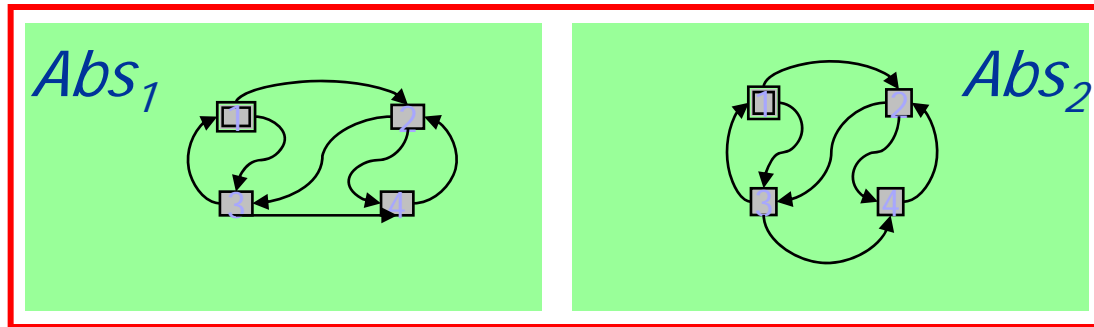
Sys



Sys₁

Sys₂

$$\frac{\begin{array}{l} Sys_1 \leq Abs_1 \\ Sys_2 \leq Abs_2 \end{array}}{Sys_1 / Sys_2 \leq Abs_1 / Abs_2}$$



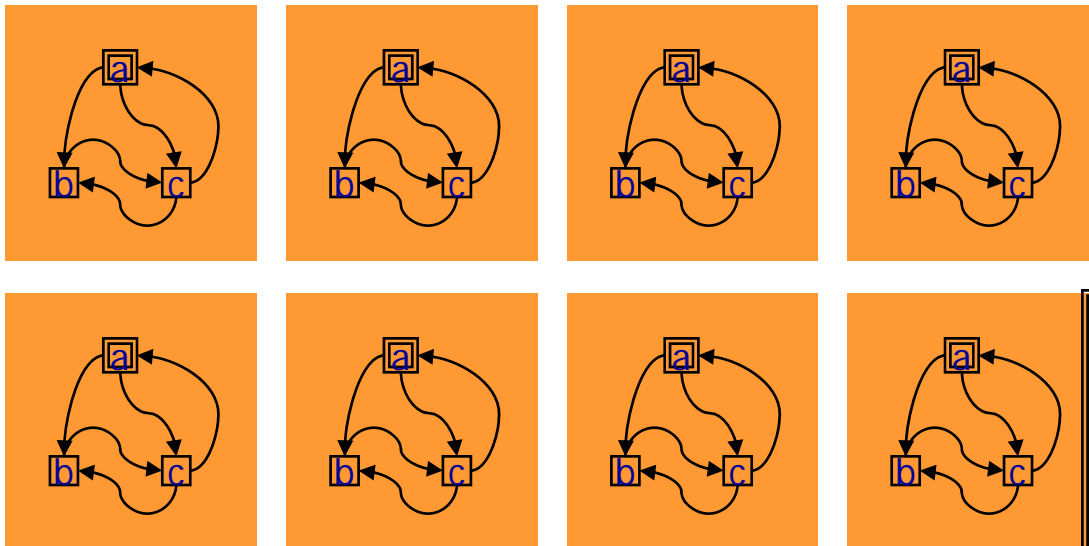
Abs₁

Abs₂

$$\frac{\begin{array}{l} Sys_1 \leq Abs_1 \\ Sys_2 \leq Abs_2 \\ Abs_1 / Abs_2 \leq Abs \end{array}}{Sys \leq Abs}$$

Abstraction & Compositionality

dealing w stateexplosion

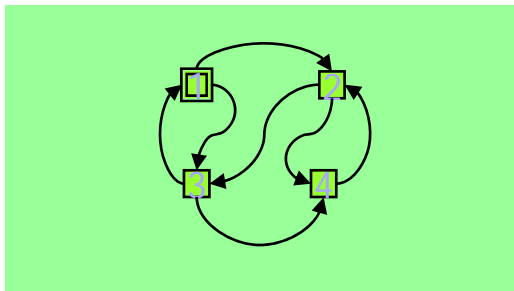
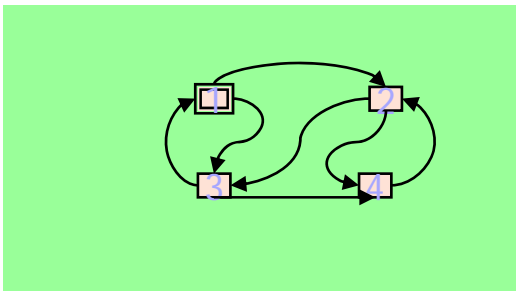


"trace" inclusion

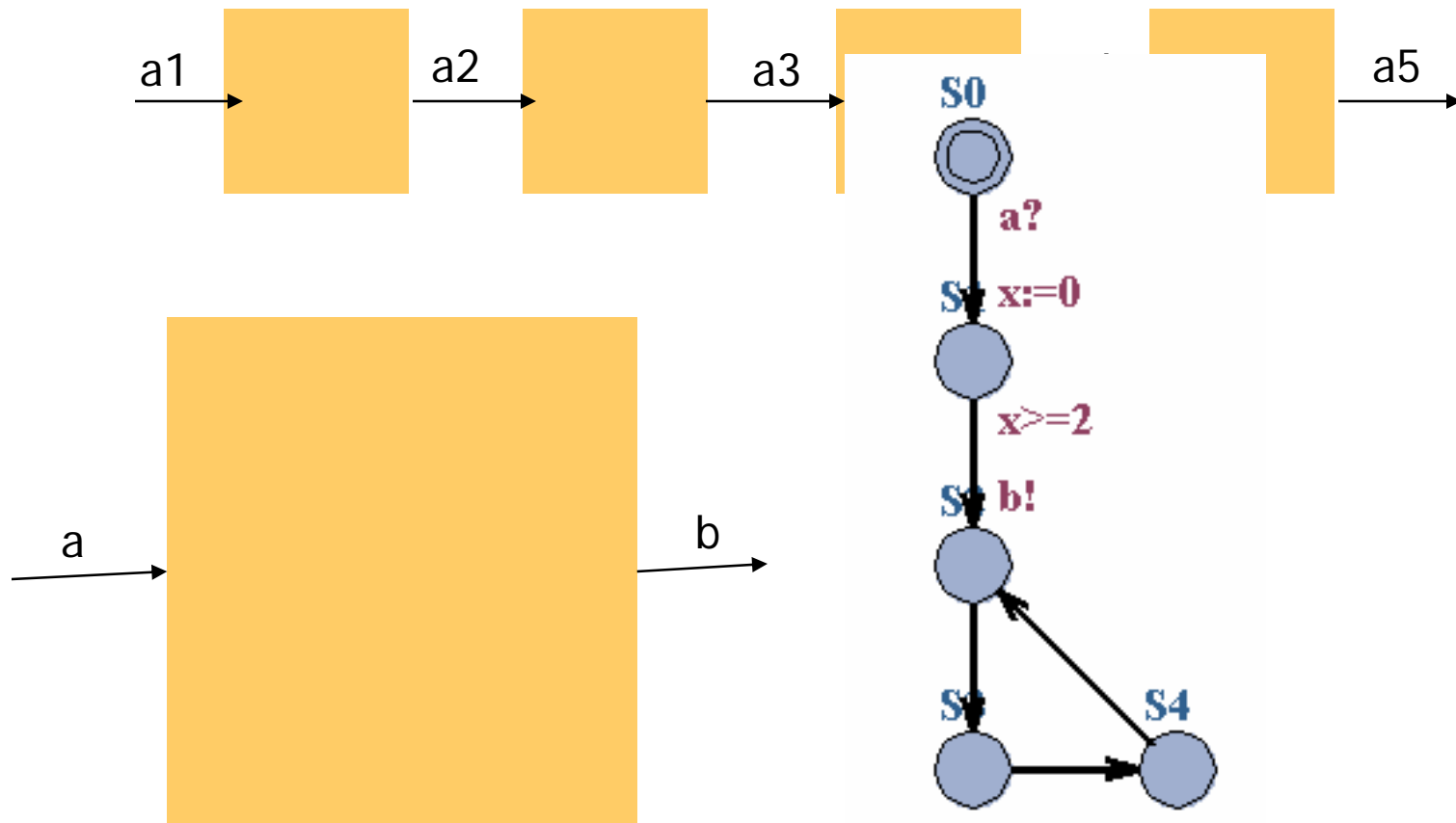
$$\frac{C_1 \prec A_1 \quad C_2 \prec A_2}{C_1|C_2 \prec A_1|A_2}$$

Concrete

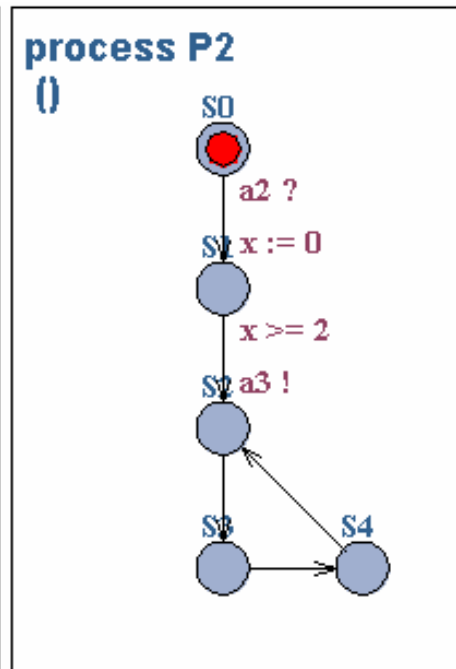
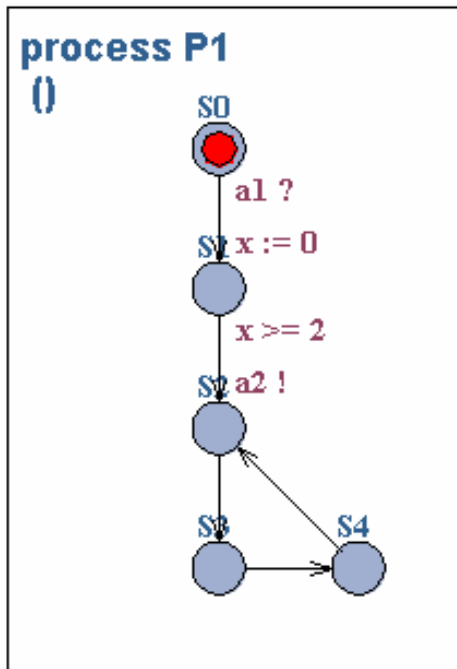
Abstract



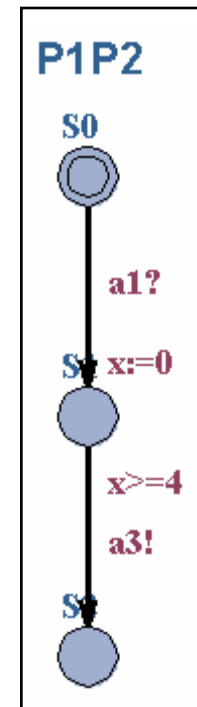
Abstraction Example



Example *Continued*



abstracted
by



Proving abstractions

using reachability

process
(

Applied to

IEEE 1394a Root contention protocol
(Simons, Stoelinga)

B&O Power Down Protocol
(Ejersbo, Larsen, Skou, FTRTFT2k)

minimizes
BAD
computations
of PoP1

$A[]$ not TestAbstPoP1.BAD

Henrik Ejersbo Jensen PhD Thesis 1999

Further Optimizations



BRICS

Basic Research
in Computer Science



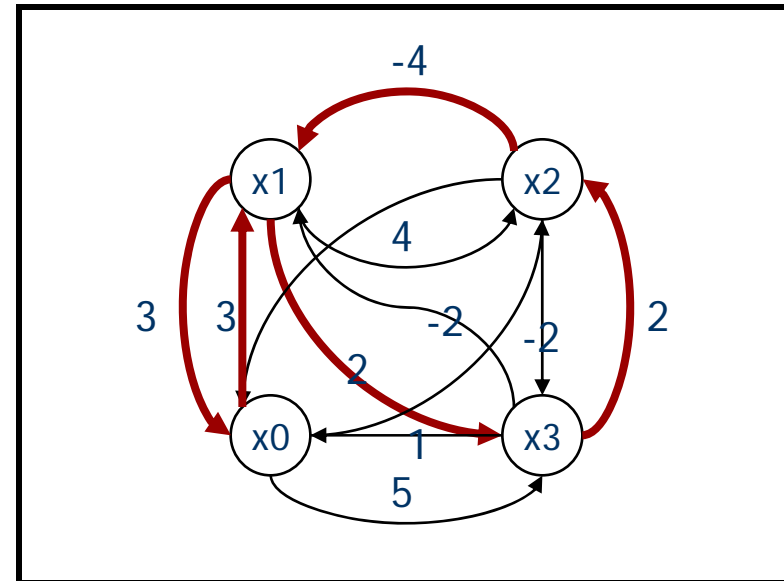
CENTER FOR INDLEJREDE SOFTWARE SYSTEMER

Datastructures for Zones

- Difference Bounded Matrices (DBMs)

- Minimal Constraint Form

[RTSS97]

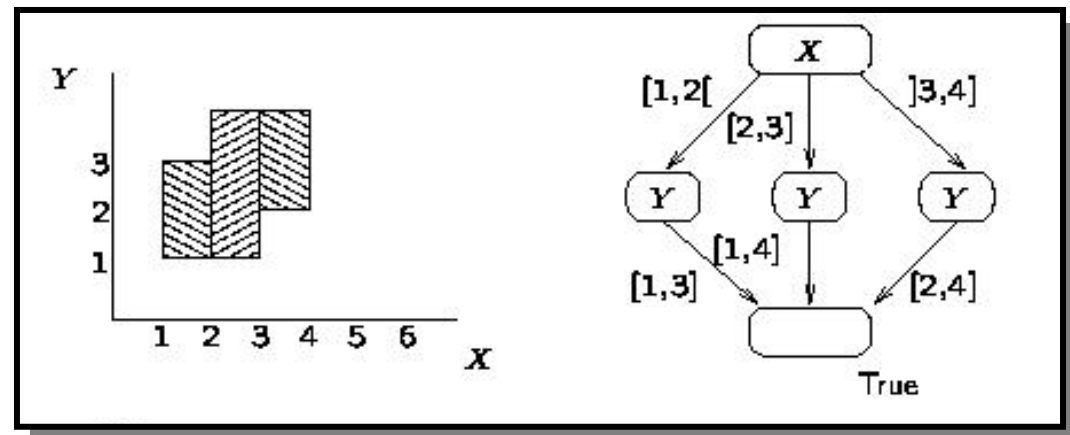


- Clock Difference Diagrams

[CAV99]

- PW List

[SPIN03]



Datastructures for Zones

■ Difference Bounded

UPPAAL DBM Library
The library used to manipulate DBMs in UPPAAL

Main Page | Download | Ruby Binding | Help | Contact us

Welcome!

DBMs [dill89, rokicki93, lpw:fct95, bengtsson02] are efficient data structures to represent clock constraints in timed automata [ad90]. They are used in UPPAAL [lpy97, by04, bd04] as the core data structure to represent time. The library features all the common operations such as up (delay, or future), down (past), general updates, different extrapolation functions, etc.. on DBMs and federations. The library also supports subtractions. The API is in C and C++. The C++ part uses active clocks and hides memory management.

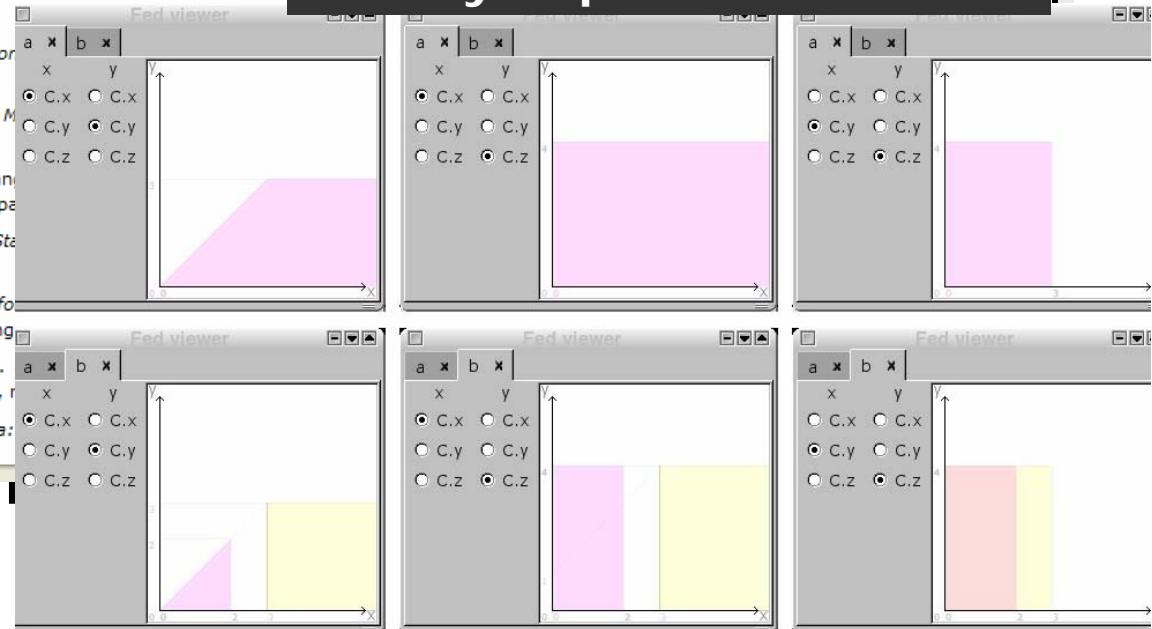
References

- [dill89] David L. Dill. *Timing Assumptions and Verification*. Springer Berlin 1989, pp 197-212.
- [rokicki93] Tomas Gerhard Rokicki. *Representing and Manipulating Difference Bound Matrices*. University 1993.
- [lpw:fct95] Kim G. Larsen, Paul Pettersson, and Wang Yi. *Fundamentals of Computation Theory 1995*, LNCS 965 pp 1-15.
- [bengtsson02] Johan Bengtsson. *Clocks, DBM, and State Space Reduction*. University 2002.
- [ad90] Rajeev Alur and David L. Dill. *Automata for Modeling and Verification*. Colloquium on Algorithms, Languages, and Programming 1990.
- [lpy97] Kim G. Larsen, Paul Pettersson, and Wang Yi. *Software Tools for Technology Transfer*, October 1997, pp 206-221.
- [by04] Johan Bengtsson and Wang Yi. *Timed Automata: Theory and Practice*. LNCS 3008 pp 1-15.



Alexandre David

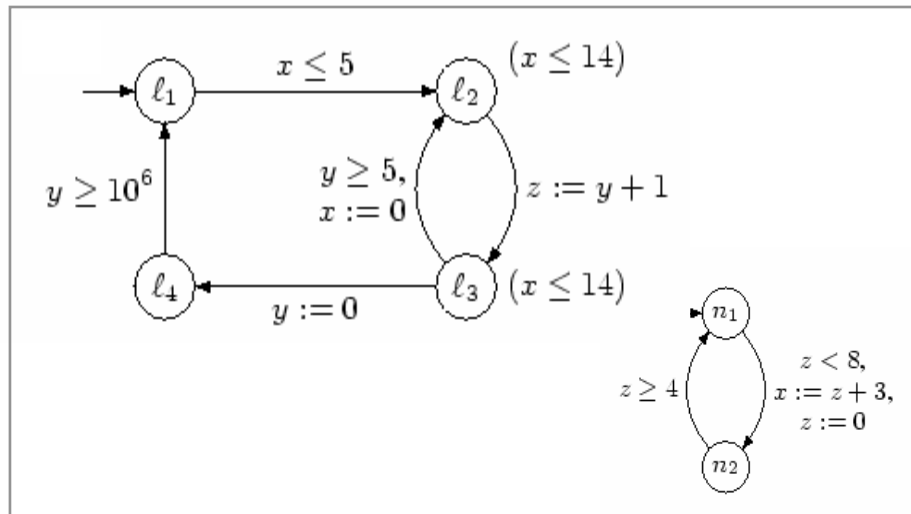
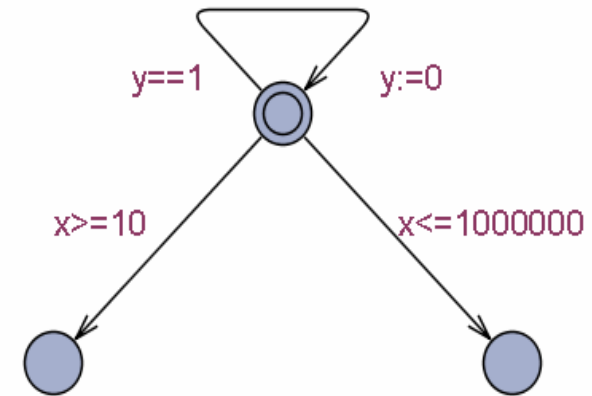
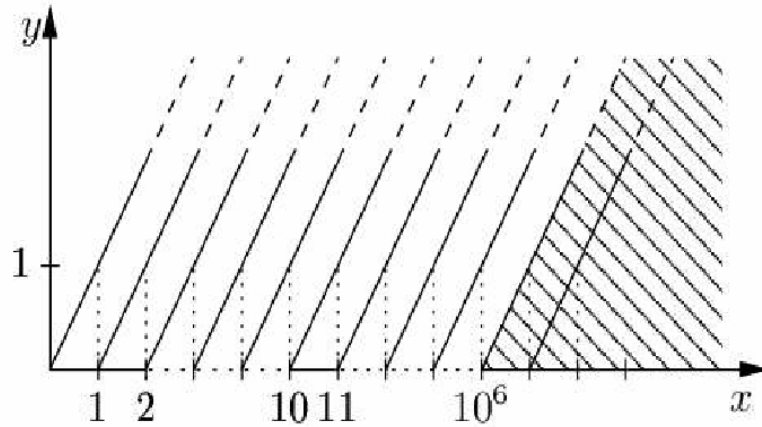
Elegant RUBY bindings for easy implementations



[SPIN03]

Zone Abstractions

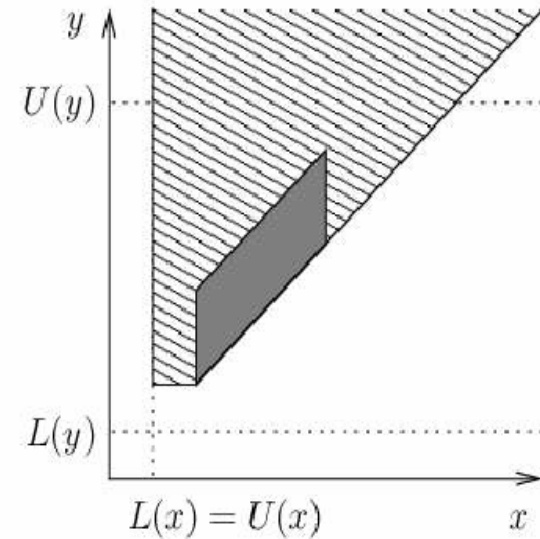
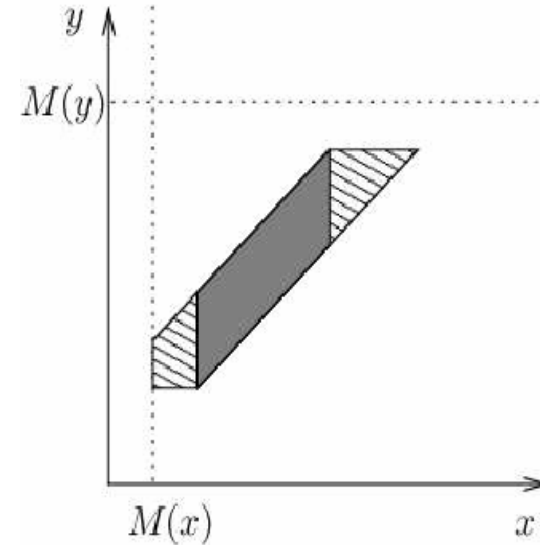
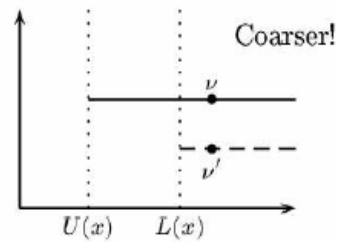
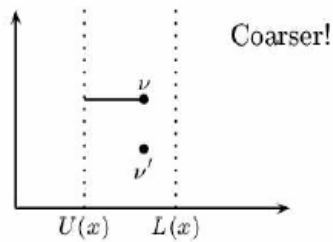
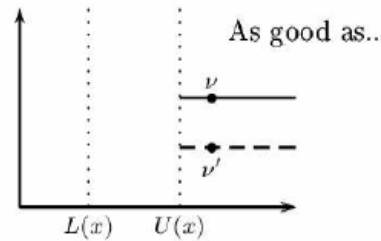
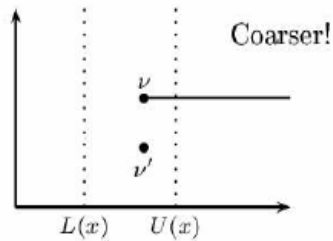
[TACAS03, TACAS04]



- Abstraction taking maximum constant into account necessary for termination
- Utilization of distinction between lower and upper bounds
- Utilization of location-dependency

LU Abstraction

[TACAS04]



THEOREM

For any state in the LU- abstraction there is a state in the original set simulating it



LU abstraction is exact wrt reachability

Zone abstractions

Model	Classical			Loc. dep. Max			Loc. dep. LU			Convex Hull		
	-n1			-n2			-n3			-A		
	Time	States	Mem	Time	States	Mem	Time	States	Mem	Time	States	Mem
f5	4.02	82,685	5	0.24	16,980	3	0.03	2,870	3	0.03	3,650	3
f6	597.04	1,489,230	49	6.67	158,220	7	0.11	11,484	3	0.10	14,658	3
f7				352.67	1,620,542	46	0.47	44,142	3	0.45	56,252	5
f8							2.11	164,528	6	2.08	208,744	12
f9							8.76	598,662	19	9.11	754,974	39
f10							37.26	2,136,980	68	39.13	2,676,150	143
f11							152.44	7,510,382	268			
c5	0.55	27,174	3	0.14	10,569	3	0.02	2,027	3	0.03	1,651	3
c6	19.39	287,109	11	3.63	87,977	5	0.10	6,296	3	0.06	4,986	3
c7				195.35	813,924	29	0.28	18,205	3	0.22	14,101	4
c8							0.98	50,058	5	0.66	38,060	7
c9							2.90	132,623	12	1.89	99,215	17
c10							8.42	341,452	29	5.48	251,758	49
c11							24.13	859,265	76	15.66	625,225	138
c12							68.20	2,122,286	202	43.10	1,525,536	394
bus	102.28	6,727,443	303	66.54	4,620,666	254	62.01	4,317,920	246	45.08	3,826,742	324
philips	0.16	12,823	3	0.09	6,763	3	0.09	6,599	3	0.07	5,992	3
sched	17.01	929,726	76	15.09	700,917	58	12.85	619,351	52	55.41	3,636,576	427

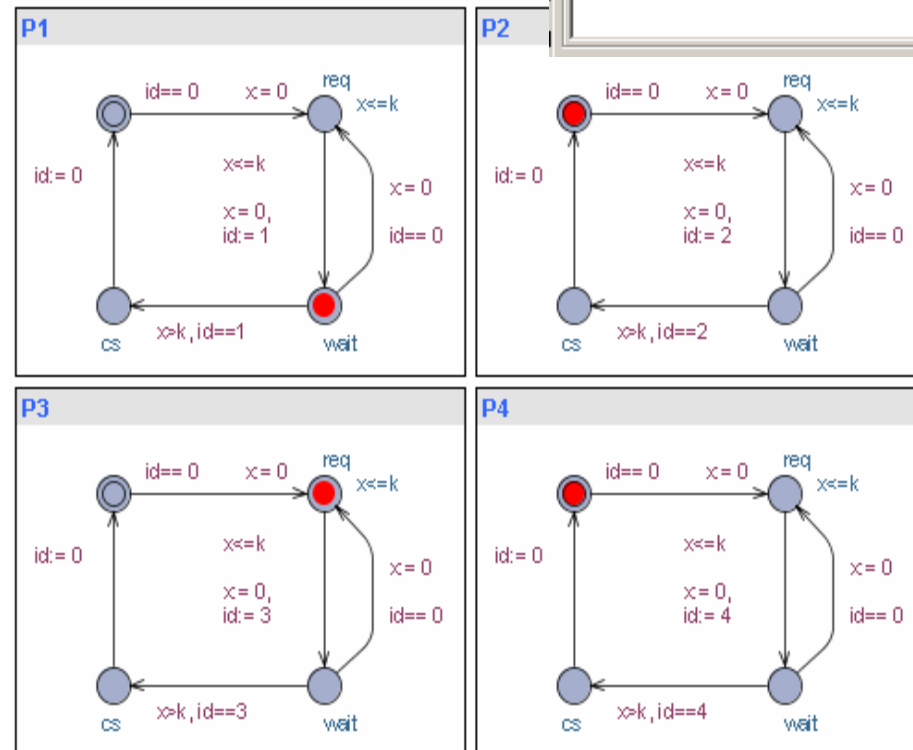
Symmetry Reduction

- Exploitation of full symmetry may give factorial reduction
- Many timed systems are inherently symmetric
- Computation of canonical state representative using swaps.

[Formats 2003]

```

Variables
id = 1
P1.x in [0,2]
P3.x in [0,2]
P2.x - P1.x in [0,inf]
P3.x - P1.x in [0,2]
P4.x - P1.x in [0,inf]
P3.x - P2.x in [-inf,0]
P4.x = P2.x
P4.x - P3.x in [0,inf]
    
```



Symmetry Reduction

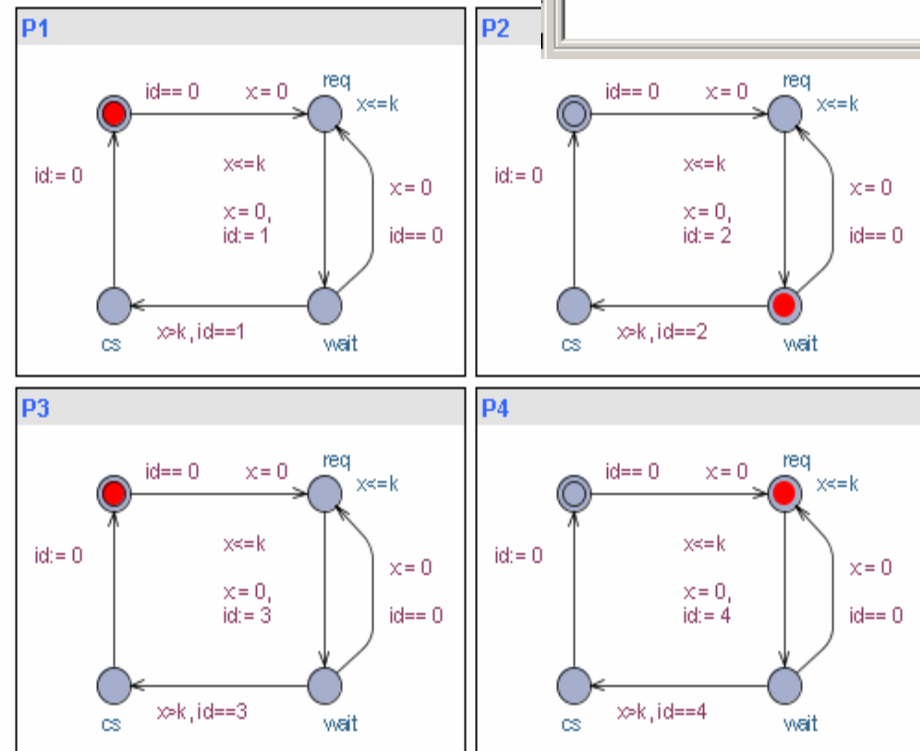
- Exploitation of full symmetry may give factorial reduction
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- Computation of canonical state representative using swaps.

[Formats 2003]

SWAP: 1→2 ; 3→4

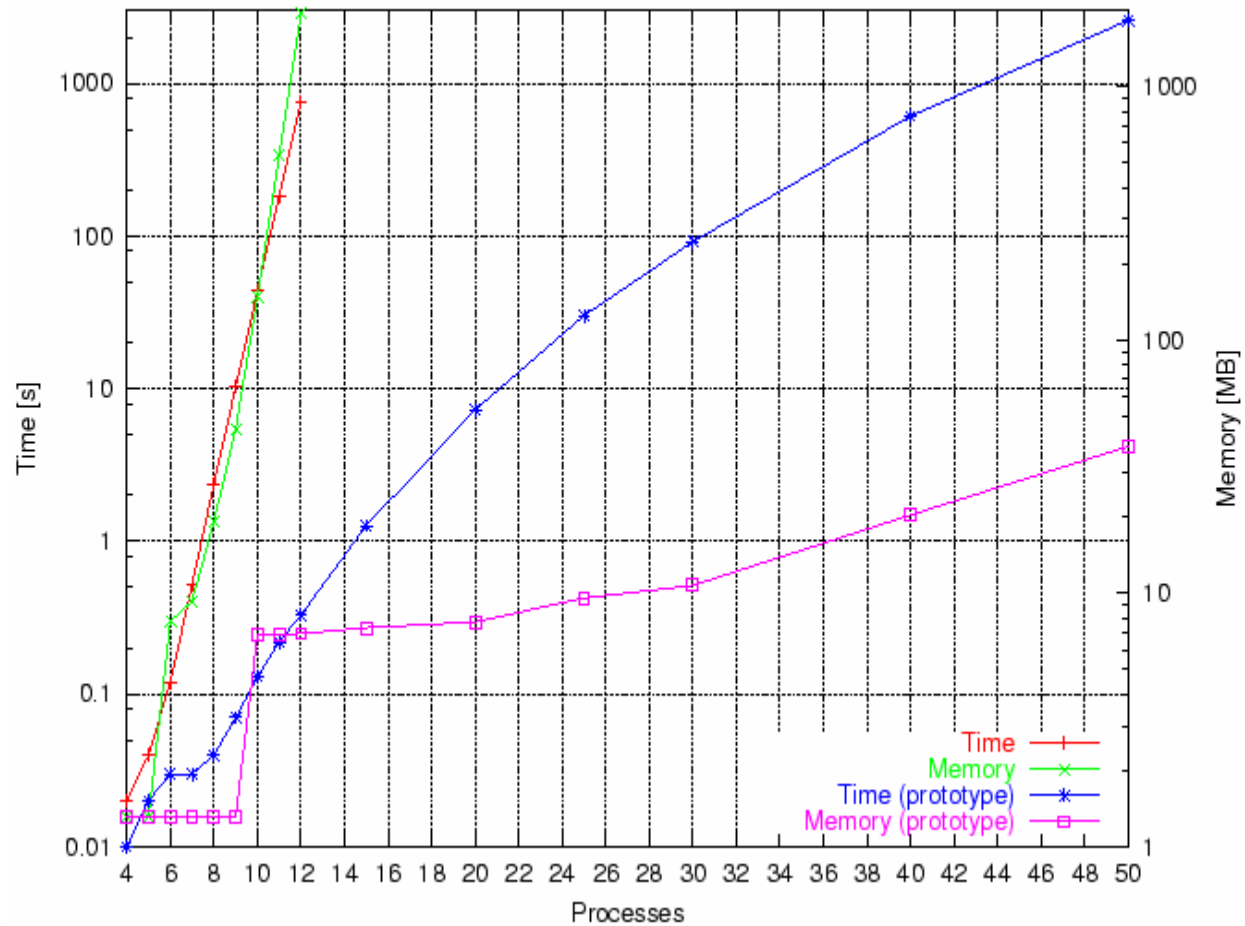
```

Variables
-----
id = 2
P2.x in [0,2]
P4.x in [0,2]
P2.x - P1.x in [-inf,0]
P3.x = P1.x
P4.x - P1.x in [-inf,0]
P3.x - P2.x in [0,inf]
P4.x - P2.x in [0,2]
P4.x - P3.x in [-inf,0]
    
```



Symmetry Reduction

[Formats 2003]



Symmetry Reduction

UPPAAL 3.6

- Iterators `for (i: int[0,4]) { }`
- Quantifiers `forall (i: int[0,4]) a[i]==0`
- Selection `select i: int[0,4]; guard...`
- Template sets `process P[4](...) { }`
- Scalar set based symmetry reduction
- Compact state-space representations
- Priorities

Gerd Behrmann

4 6 8 10 12 14 16 18 20 22

Martijn Henriks, Nijmegen U

Processes

D-UPPAAL

Gerd Behrmann

- Distributed implementation of UPPAAL on PC-cluster [CAV'00, PDMC'02, STTT'03].
- WWW frontend available
- Applications
 - Synthesis of Dynamic Voltage Scaling strategies (CISS).
 - Ad-hoc mobile real-time protocol (Leslie Lamport) - 25GB in 3 min!
- Running on NorduGrid.
Local cluster: 50 CPUs and 50GB of RAM
- To be used as inspiration for verification GRID platform within ARTIST2 NoE.



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Gerd Behrman



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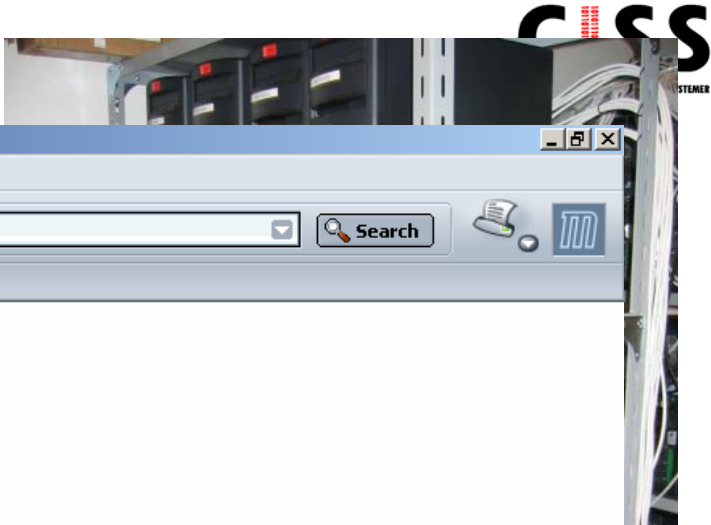
Grid Monitor

2004-11-22 CET 12:46:48

Processes: ■ Grid ■ Local

Country	Site	CPUs	Load (processes: Grid+local)	Queueing
Australia	Alfred (UniMelb)	90	7+24	9+1
	Atlas (UniMelb)	24	0+1	0+0
	Charm (UniMelb)	20	0+0	0+0
Denmark	Aalborg Grid Gateway	50	13+0	0+0
	DistLab (DIKU)	10	0+0	0+0
	HEPAX1	1	0+0	0+0
	Morpheus	18	0+0	0+0
Estonia	CMS Production server	5	0+0	0+0
	CMS test cluster	1	0+0	0+0
	EENet Kriit	6	6+0	4+0
	UT Chemistry	17	0+17	0+3
	UT CS Antarctica Clus>	20	0+0	0+0
	UT IMCB Anakonda clus>	16	0+0	0+0
Finland	UT Physics Cluster	0		0+0
	Alpha (HIP)	1	0+0	0+0
	CSC Kirppu	1	0+0	0+0
	CSC Lude	8	0+0	0+0
	Mill (Physicum)	56	0+18	8+0
	Testbed0 (HIP)	1	0+0	0+0
Germany	FZK cluster	1060	76+426	0+0
	LRZ cluster	260	0+244	0+899
Norway	Bergen Grid Cluster	4	0+0	0+0
	Oslo Grid Cluster	43	0+14	0+1
	Oslo Temp Cluster	7	0+0	0+0
	Parallell IBM Cluster	58	0+3	0+0

D-UPPAAL



File information:
 Model: Browse...
 Query: Browse...

Model checking options
 Search order: breadth first width first
 State space reduction: none conservative aggressive
 State space representation: DBM compact data structure under approximation over approximation
 New syntax: no yes

Distribution options
 Number of CPUs: 1 5 10 15 20 25 30 35 49

Run options
 Max walltime (minutes): 1 5 15 30 60 120 240

Contact information
 Email:
 Submit Query Reset

ueueing
+1
+0
+0
+0
+0
+0
+0
+0
+0
+0
+3
+0
+0
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+0
+0
+0
+0
+899
+0
+0
+1
+0
+0