Introduction to Artificial Intelligence

Problem Solving and Search

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Outline

- **Problem solving**
- **Problem types**
- **Problem formulation**
- **Example problems**
- **Basic search algorithms**

Problem solving

Offline problem solving

Acting only with complete knowledge of problem and solution

Online problem solving

Acting without complete knowledge

Here

Here we are concerned with offline problem solving only

Example: Travelling in Romania

Scenario

On holiday in Romania; currently in Arad

Flight leaves tomorrow from Bucharest

Example: Travelling in Romania

Scenario

On holiday in Romania; currently in Arad
Flight leaves tomorrow from Bucharest

Goal

Be in Bucharest

Example: Travelling in Romania

Scenario

On holiday in Romania; currently in Arad

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Goal

Be in Bucharest

Formulate problem

States: various cities

Actions: drive between cities

Example: Travelling in Romania

Scenario

On holiday in Romania; currently in Arad
Flight leaves tomorrow from Bucharest

Goal

Be in Bucharest

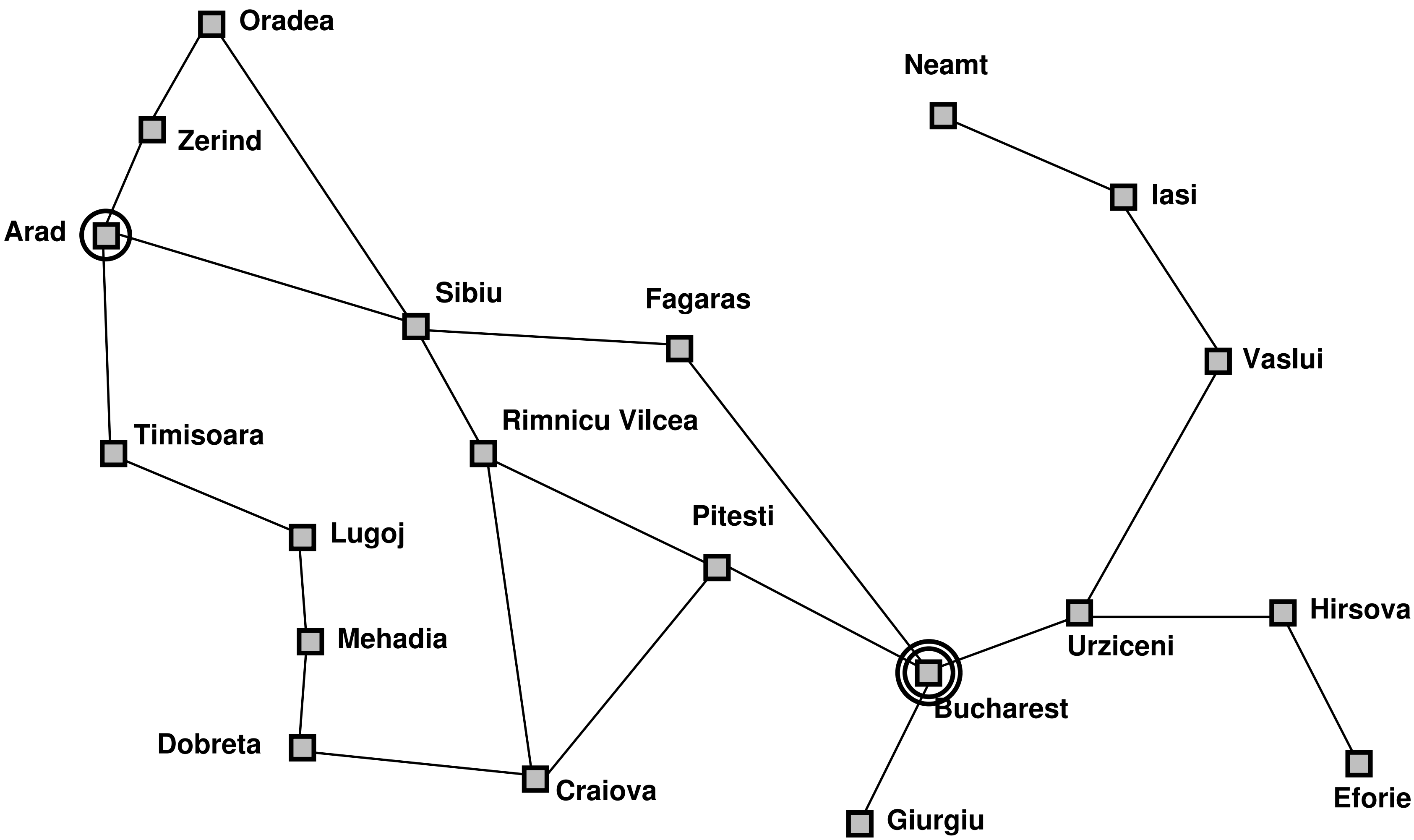
Formulate problem

States: various cities
Actions: drive between cities

Solution

Appropriate sequence of cities
e.g.: Arad, Sibiu, Fagaras, Bucharest

Example: Travelling in Romania



Problem types

Single-state problem

- observable (at least the initial state)
- deterministic
- static
- discrete

Multiple-state problem

- partially observable (initial state not observable)
- deterministic
- static
- discrete

Contingency problem

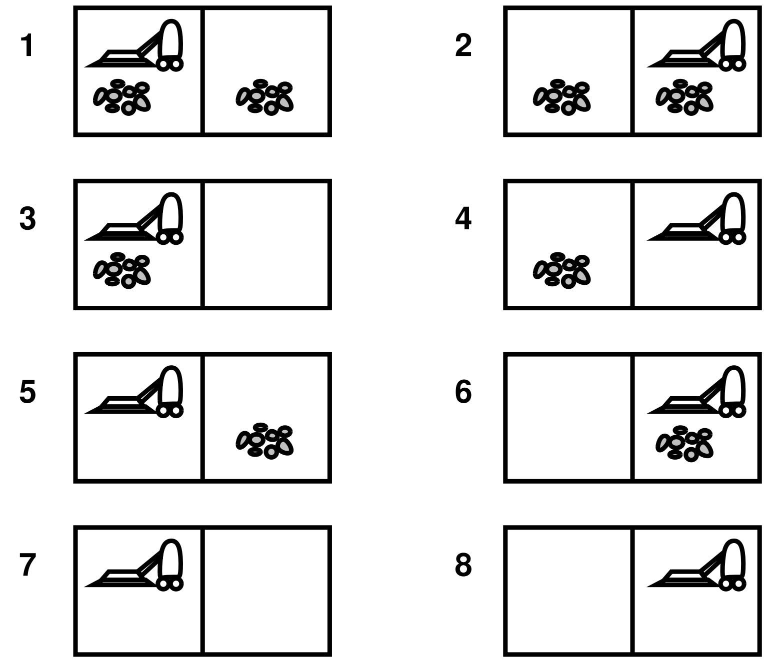
- partially observable (initial state not observable)
- non-deterministic

Example: vacuum-cleaner world

Single-state

Start in: 5

Solution:

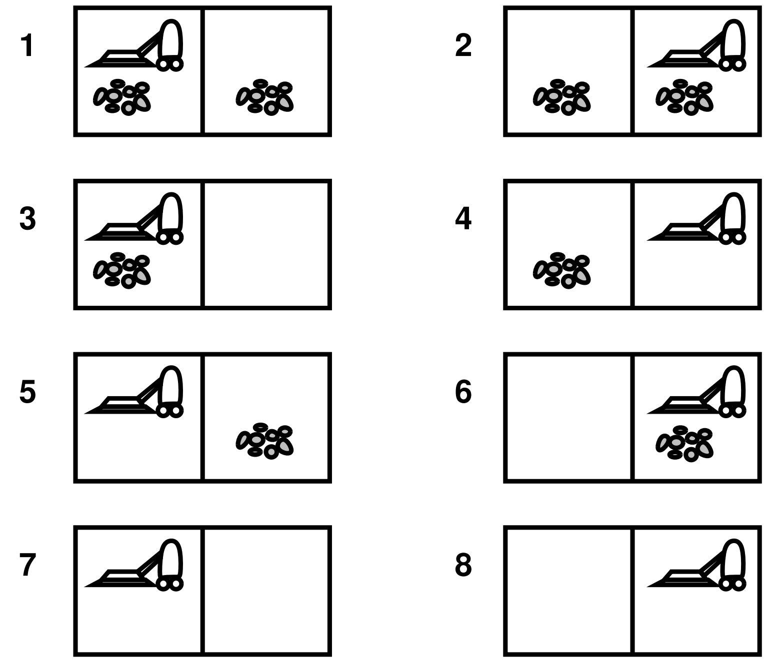


Example: vacuum-cleaner world

Single-state

Start in: 5

Solution: [*right*, *suck*]



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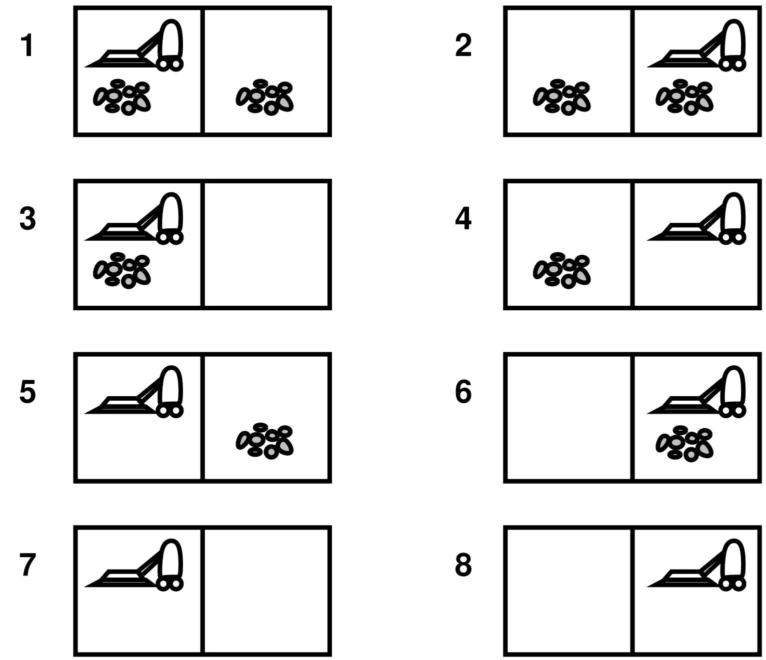


Single-state

Start in: 5

Solution: **[*right, suck*]**

Multiple-state

Start in: $\{1, 2, 3, 4, 5, 6, 7, 8\}$

Solution:

Example: vacuum-cleaner world

Single-state

Start in: 5

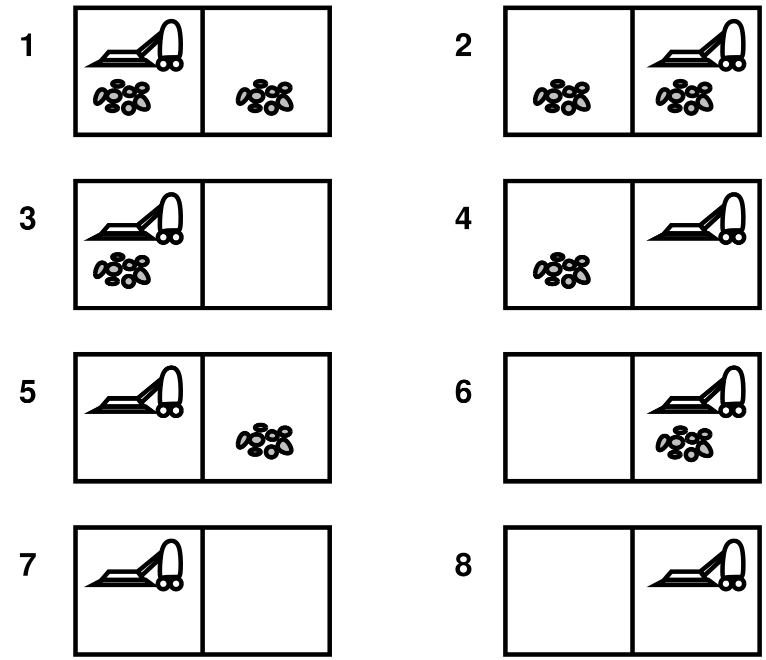
Solution: [*right*, *suck*]

Multiple-state

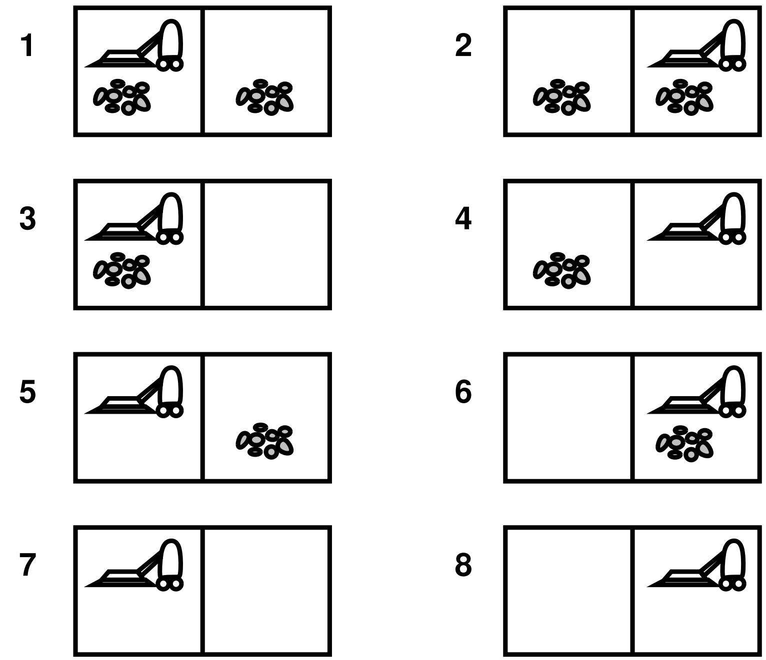
Start in: $\{1,2,3,4,5,6,7,8\}$

Solution: [*right*, *suck*, *left*, *suck*]

right	$\longrightarrow \{2,4,6,8\}$
suck	$\longrightarrow \{4,8\}$
left	$\longrightarrow \{3,7\}$
suck	$\longrightarrow \{7\}$



Example: vacuum-cleaner world



Contingency

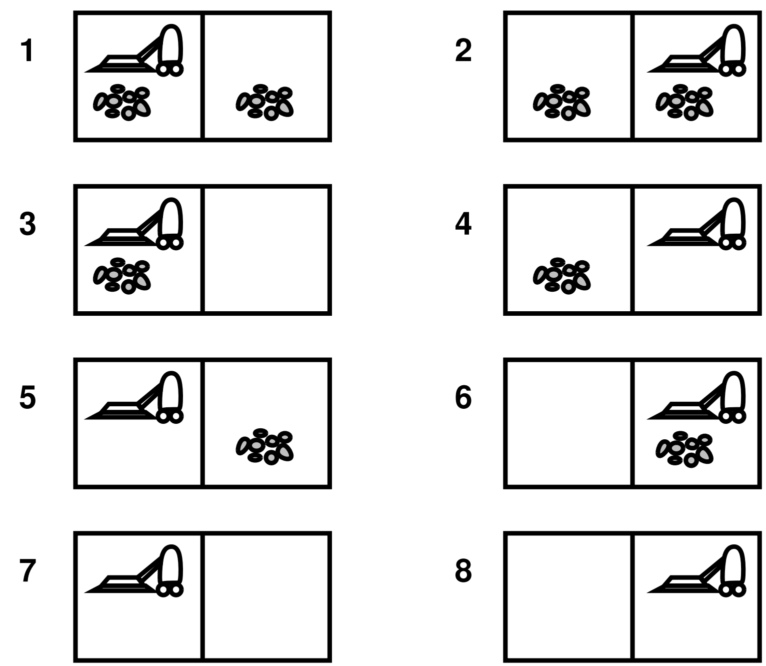
Murphy's Law:
suck **can dirty a clean carpet**

Local sensing:
dirty*/*not dirty **at location only**

Start in: $\{1, 3\}$

Solution:

Example: vacuum-cleaner world



Contingency

Murphy's Law:
***suck* can dirty a clean carpet**

Local sensing:
***dirty*/*not dirty* at location only**

Start in: $\{1, 3\}$

Solution: [***suck***, ***right***, ***suck***]

suck $\longrightarrow \{5, 7\}$

right $\longrightarrow \{6, 8\}$

suck $\longrightarrow \{6, 8\}$

Improvement: [***suck***, ***right***, **if *dirt* then *suck***]
(decide whether in 6 or 8 using local sensing)

Single-state problem formulation

Defined by the following four items

1. **Initial state**

   Example: ***Arad***

2. **Successor function S**

   Example: $S(\textbf{\textit{Arad}}) = \{ \langle \textbf{\textit{goZerind}}, \textbf{\textit{Zerind}} \rangle, \langle \textbf{\textit{goSibiu}}, \textbf{\textit{Sibiu}} \rangle, \ldots \}$

3. **Goal test**

   Example: $x =$ ***Bucharest*** **(explicit test)**

   noDirt(x) **(implicit test)**

4. **Path cost** **(optional)**

   Example: **sum of distances, number of operators executed, etc.**

Single-state problem formulation

Solution

A sequence of operators
leading from the initial state to a goal state

Selecting a state space

Abstraction

Real world is absurdly complex
State space must be abstracted for problem solving

(Abstract) state

Set of real states

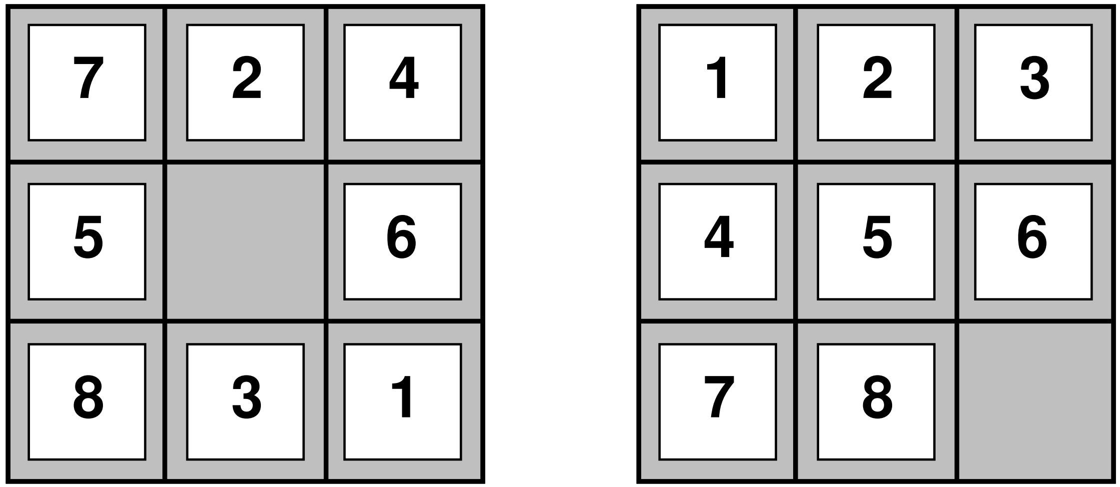
(Abstract) operator

Complex combination of real actions
Example: *Arad* $\longrightarrow$ *Zerind* represents complex set of possible routes

(Abstract) solution

Set of real paths that are solutions in the real world

Example: The 8-puzzle



Start State **Goal State**

States

Actions

Goal test

Path cost

Example: The 8-puzzle

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

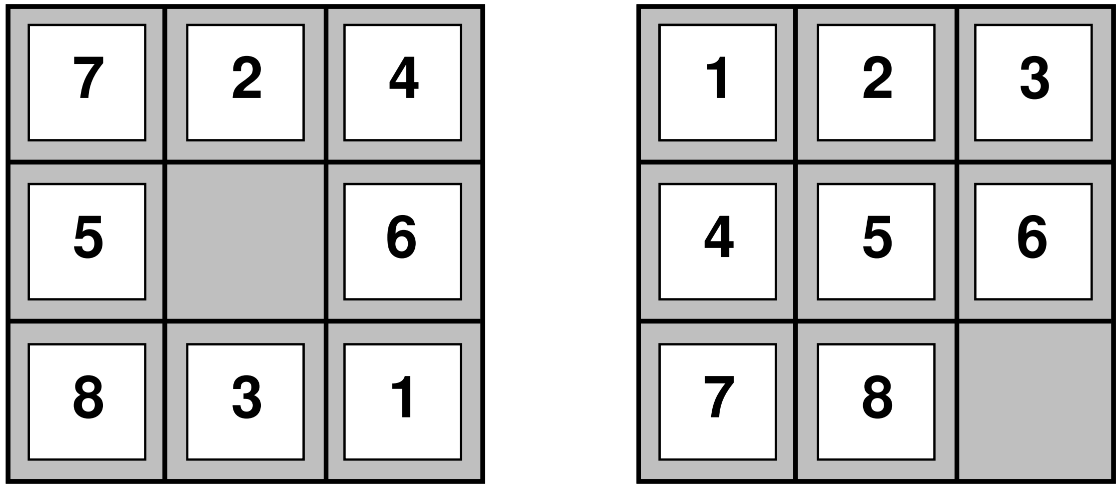
States integer locations of tiles

Actions

Goal test

Path cost

Example: The 8-puzzle



Start State

Goal State

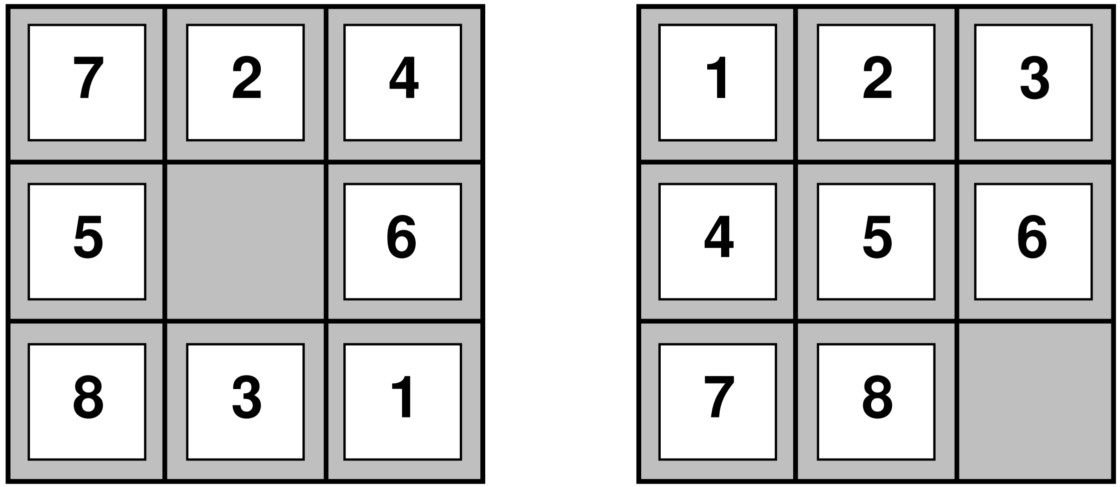
States integer locations of tiles

Actions *left*, *right*, *up*, *down*

Goal test

Path cost

Example: The 8-puzzle



Start State Goal State

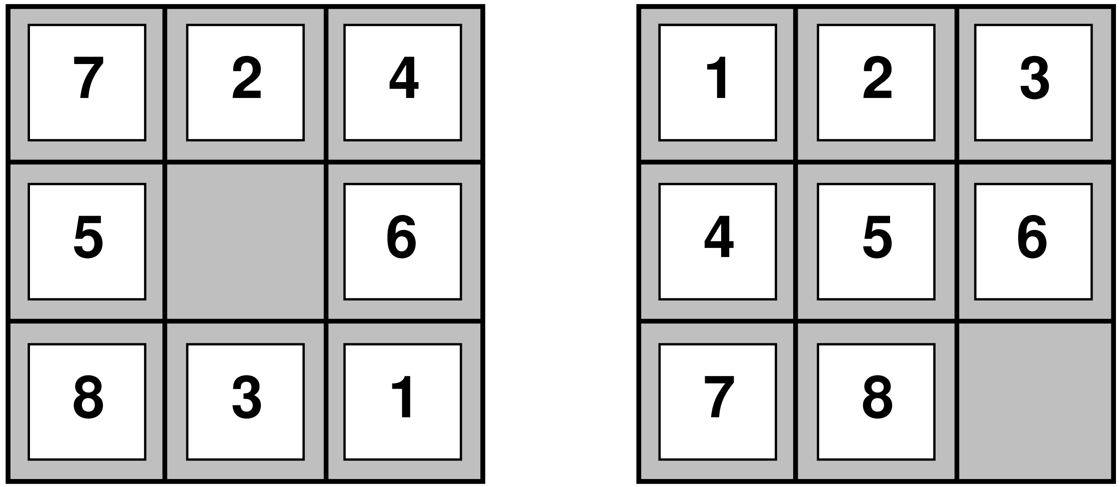
States integer locations of tiles

Actions *left*, *right*, *up*, *down*

Goal test = goal state?

Path cost

Example: The 8-puzzle



Start State   Goal State

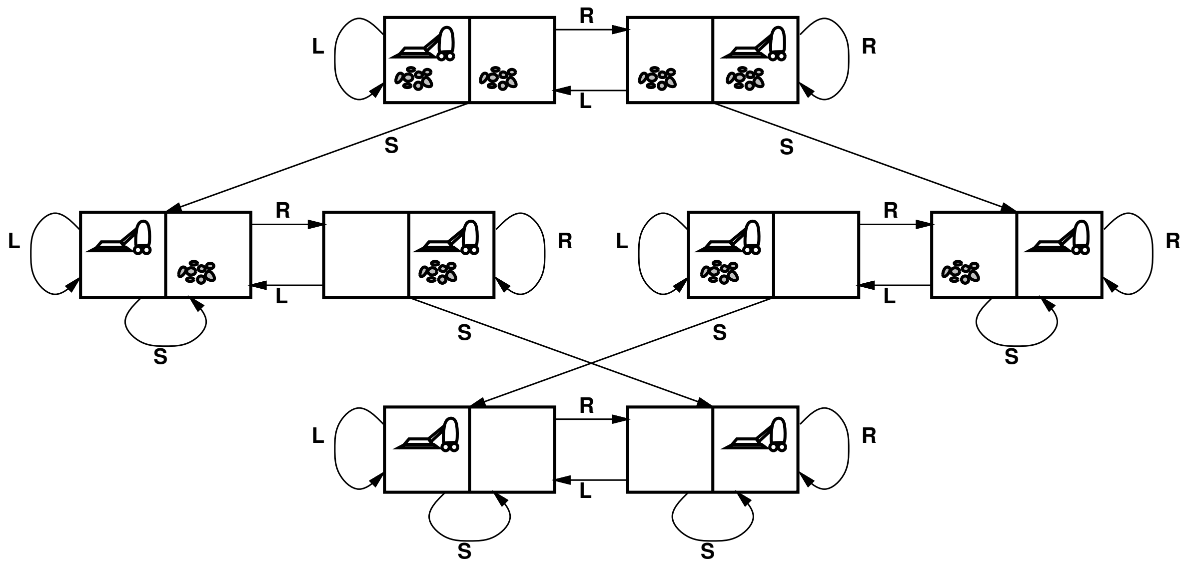
States   integer locations of tiles

Actions   *left*, *right*, *up*, *down*

Goal test   = goal state?

Path cost 1 per move

Example: Vacuum-cleaner



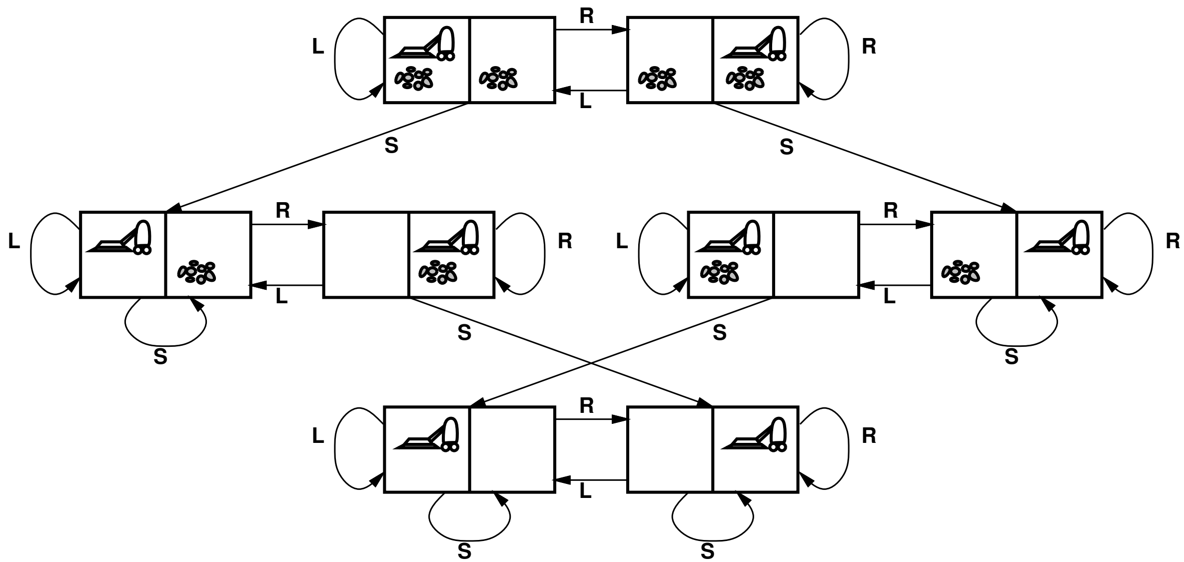
States

Actions

Goal test

Path cost

Example: Vacuum-cleaner



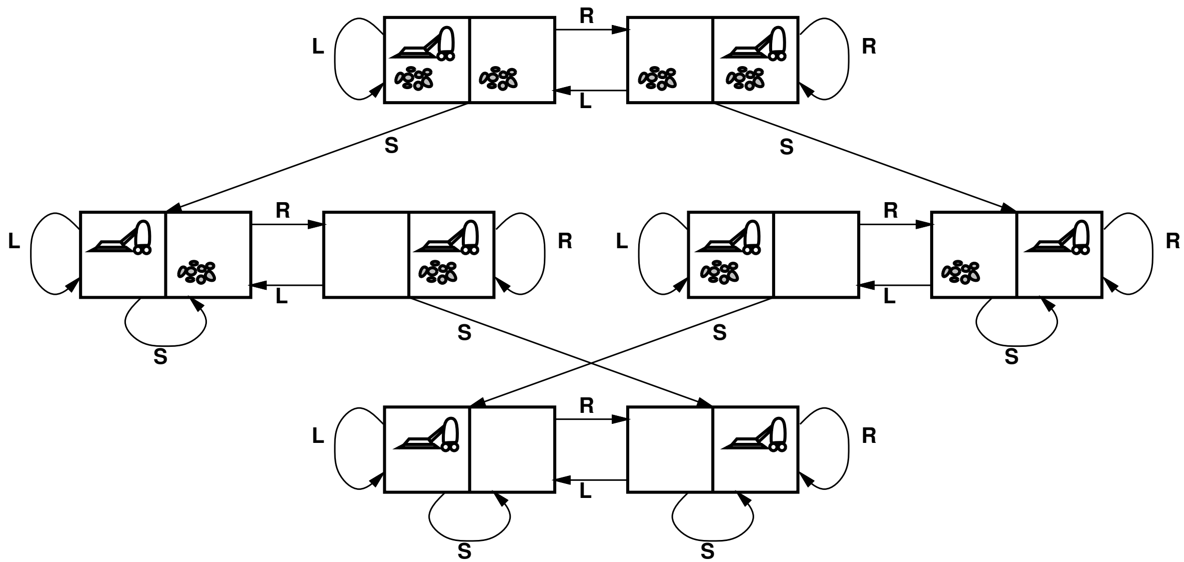
States integer dirt and robot locations

Actions

Goal test

Path cost

Example: Vacuum-cleaner



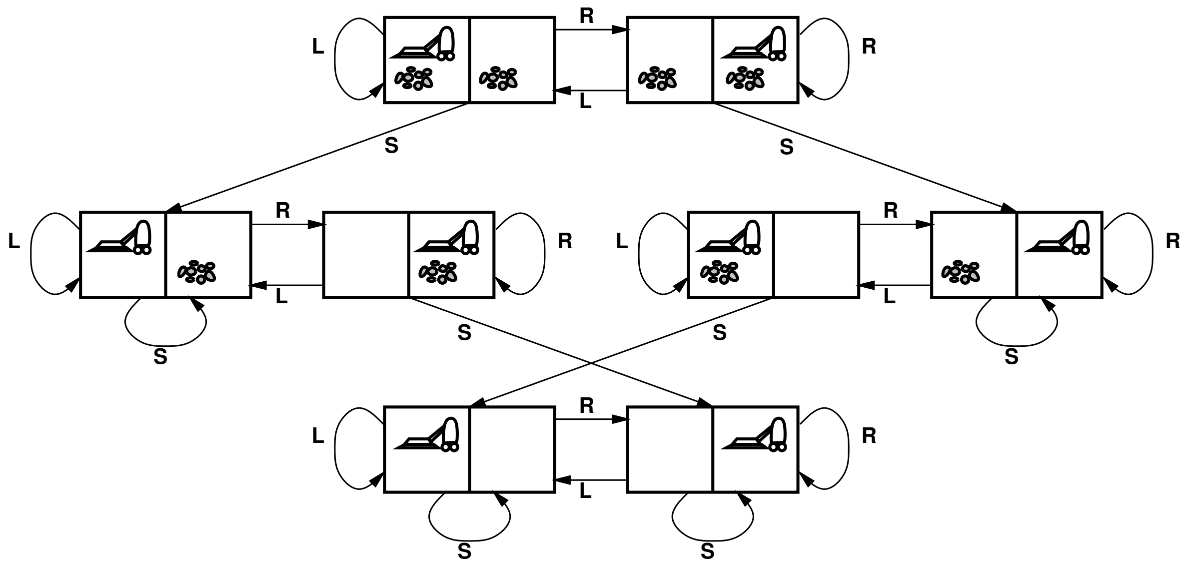
States integer dirt and robot locations

Actions *left*, *right*, *suck*, *noOp*

Goal test

Path cost

Example: Vacuum-cleaner



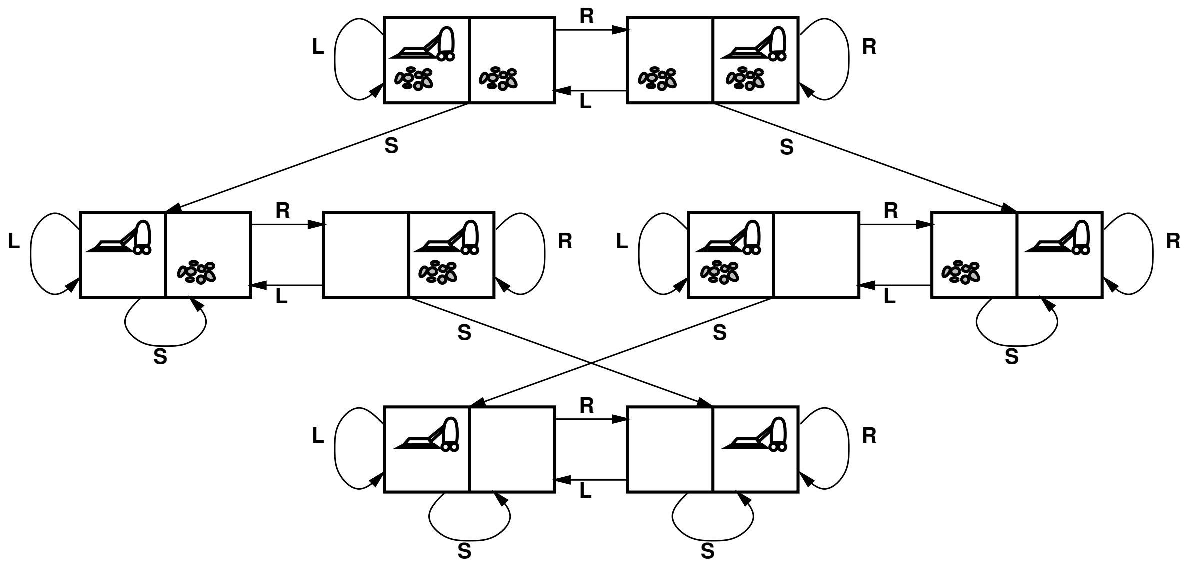
States integer dirt and robot locations

Actions ***left***, ***right***, ***suck***, ***noOp***

Goal test ***not dirty***?

Path cost

Example: Vacuum-cleaner



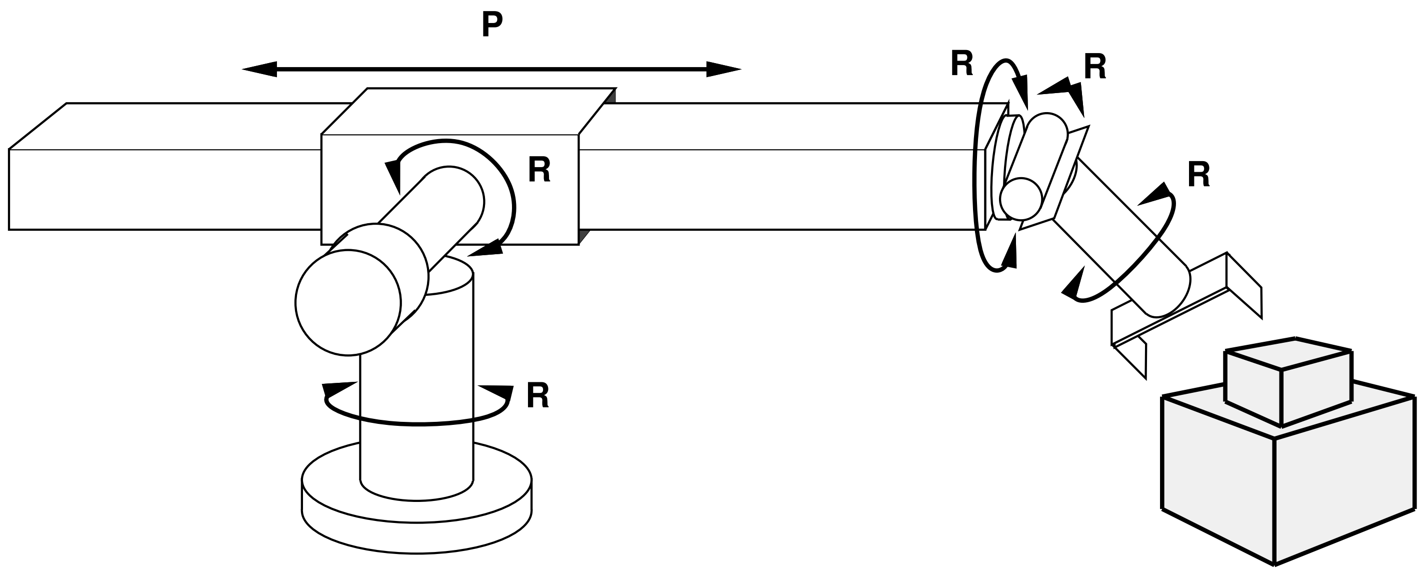
States integer dirt and robot locations

Actions ***left***, ***right***, ***suck***, ***noOp***

Goal test ***not dirty*?**

Path cost 1 **per operation** (**0 for *noOp***)

Example: Robotic assembly



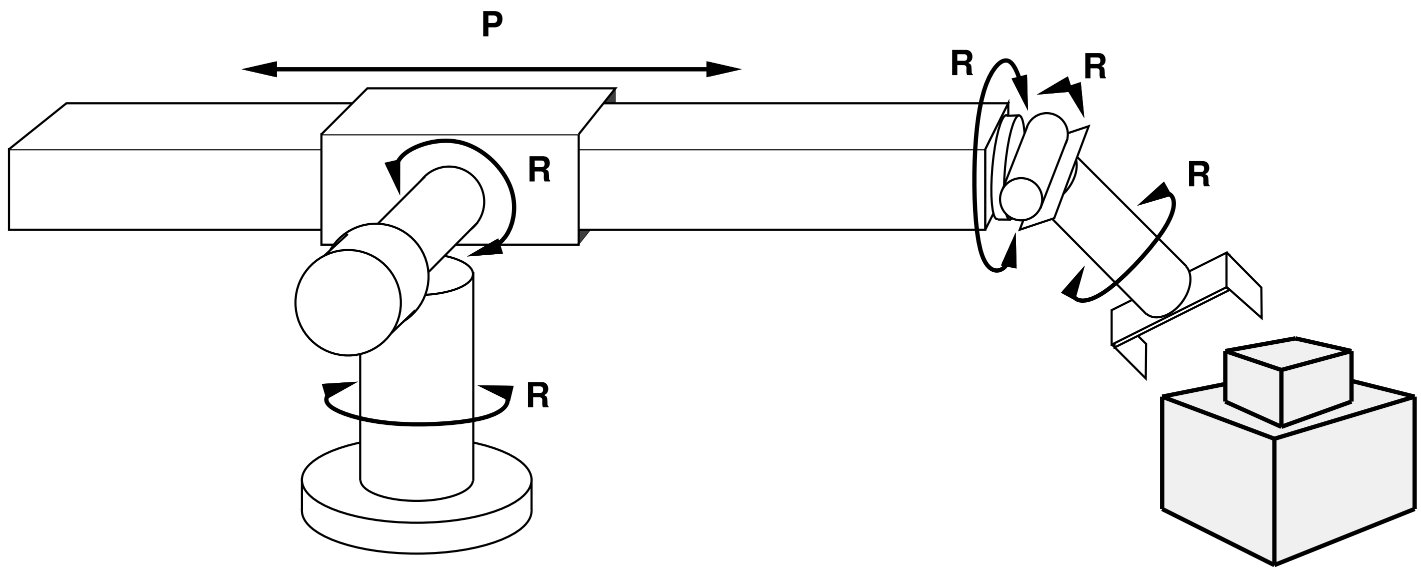
States

Actions

Goal test

Path cost

Example: Robotic assembly



States real-valued coordinates of

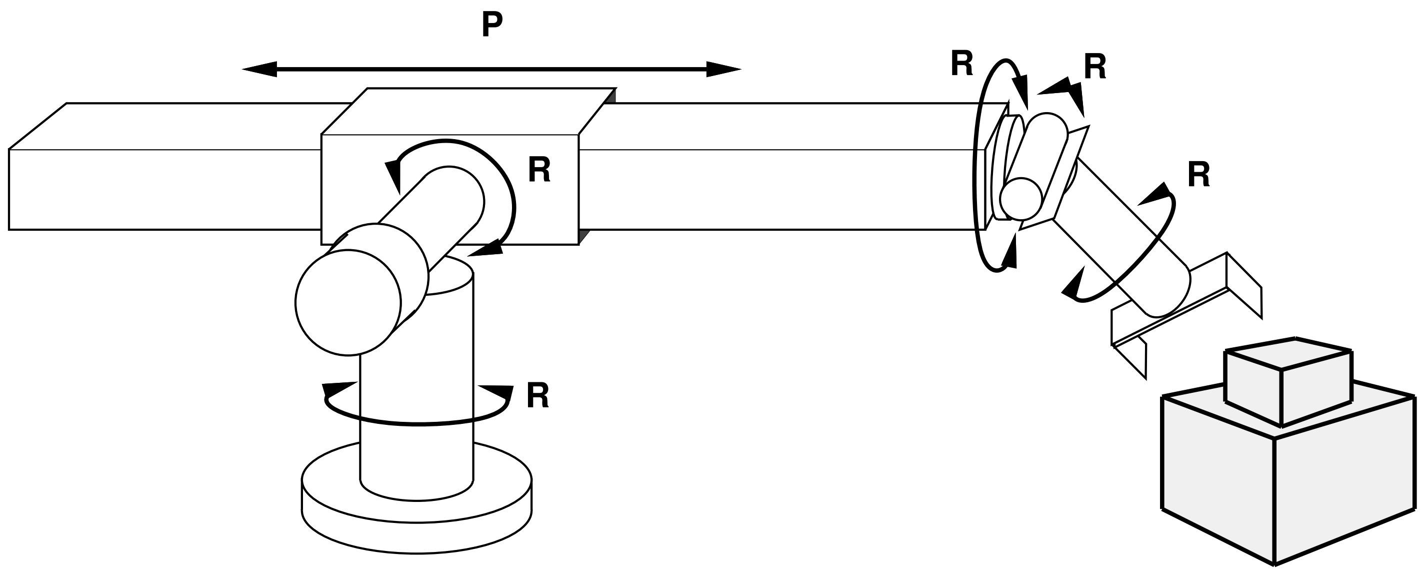
robot joint angles and parts of the object to be assembled

Actions

Goal test

Path cost

Example: Robotic assembly



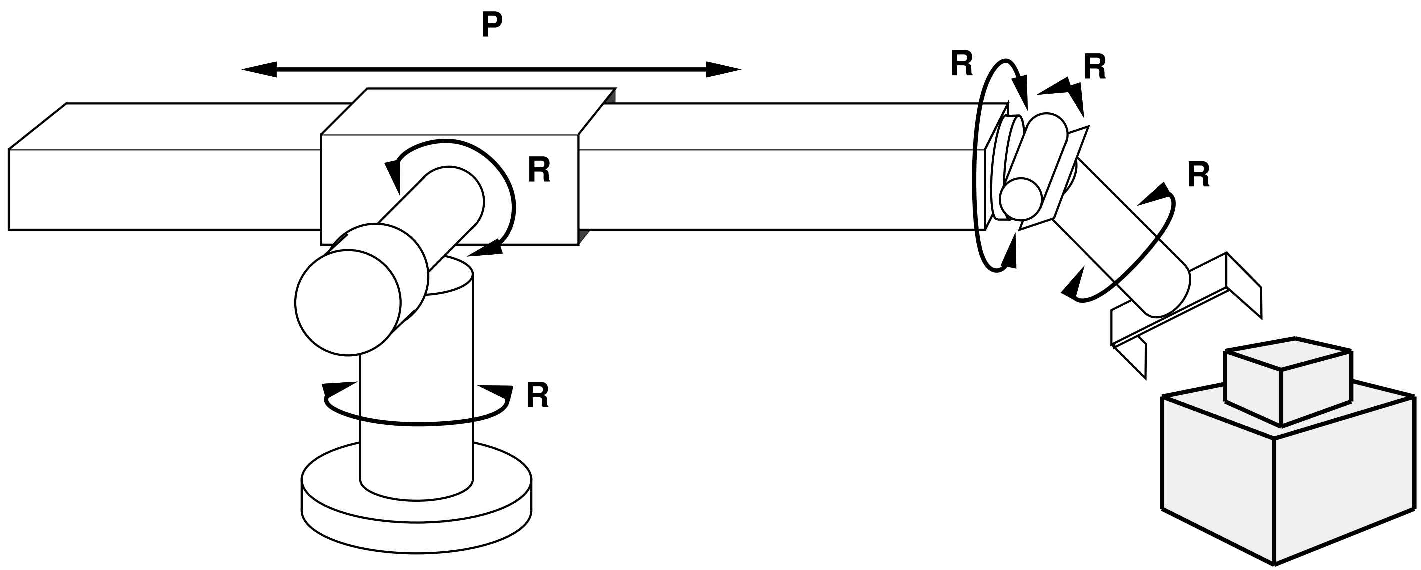
States real-valued coordinates of
robot joint angles and parts of the object to be assembled

Actions continuous motions of robot joints

Goal test

Path cost

Example: Robotic assembly



States real-valued coordinates of
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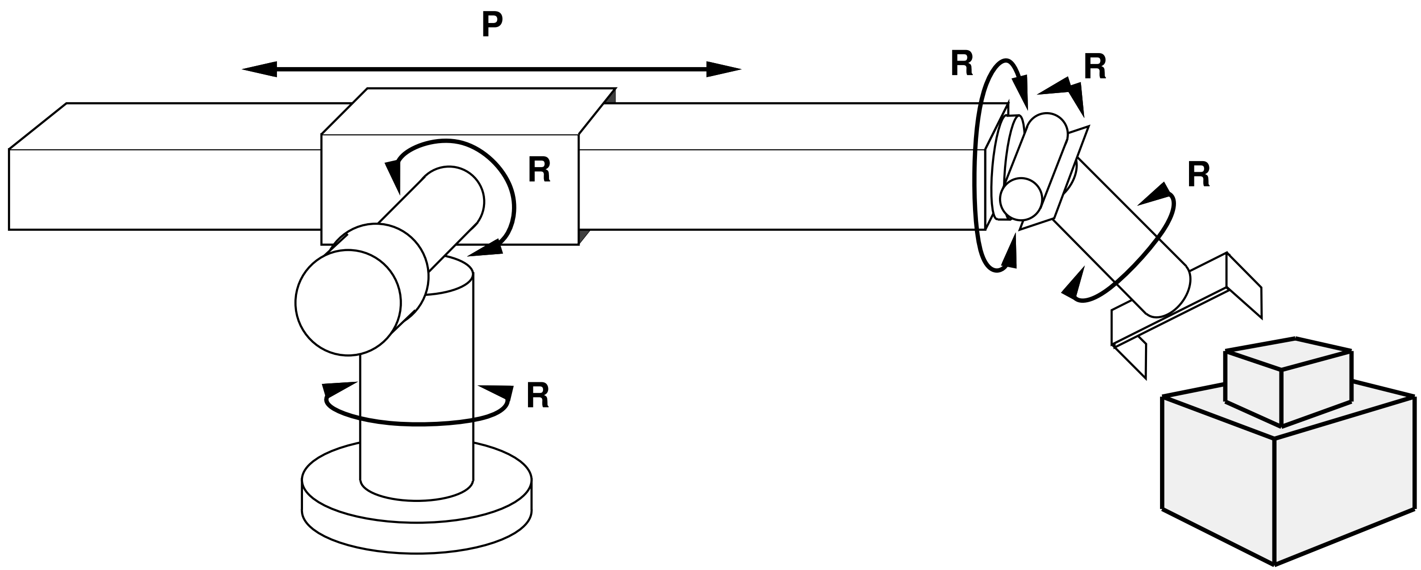
Actions continuous motions of robot joints

Goal test assembly complete?

Path cost

Example: Robotic assembly



States real-valued coordinates of
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Actions continuous motions of robot joints

Goal test assembly complete?

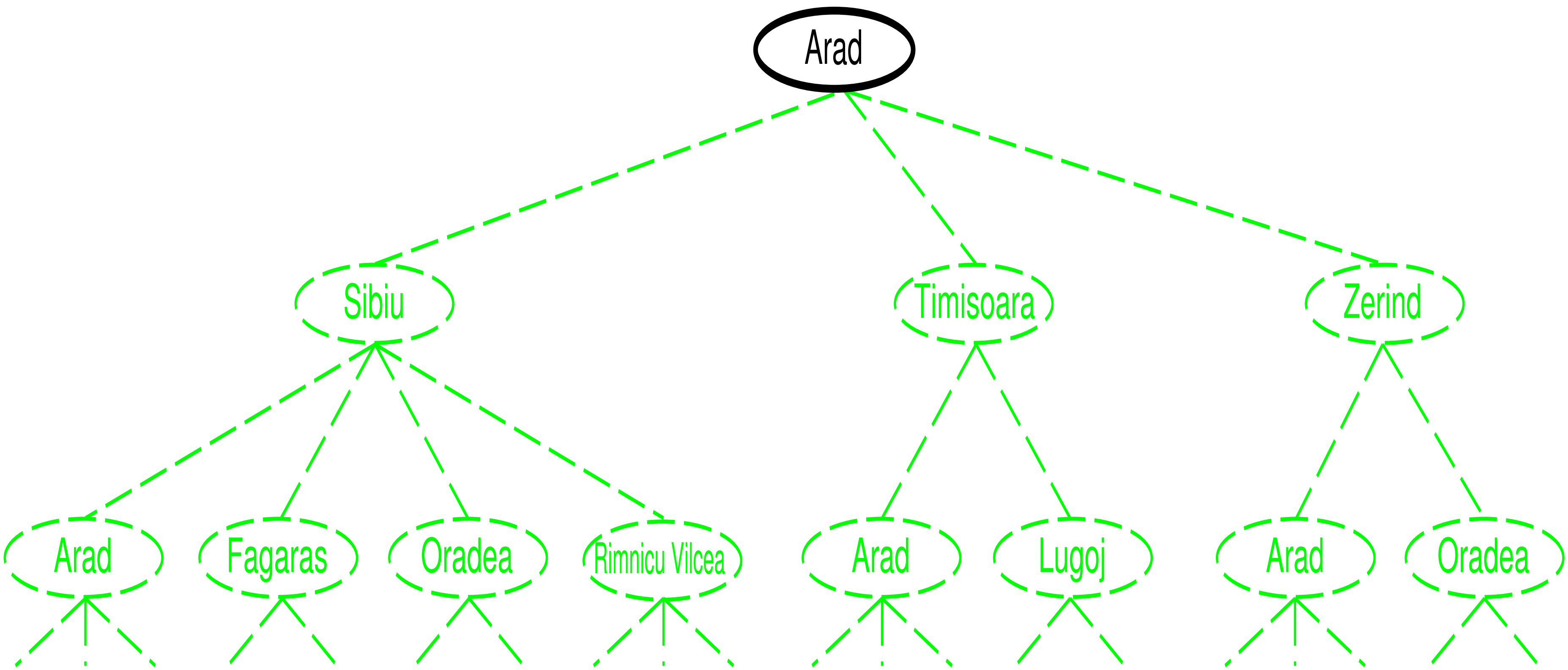
Path cost time to execute

Tree search algorithms

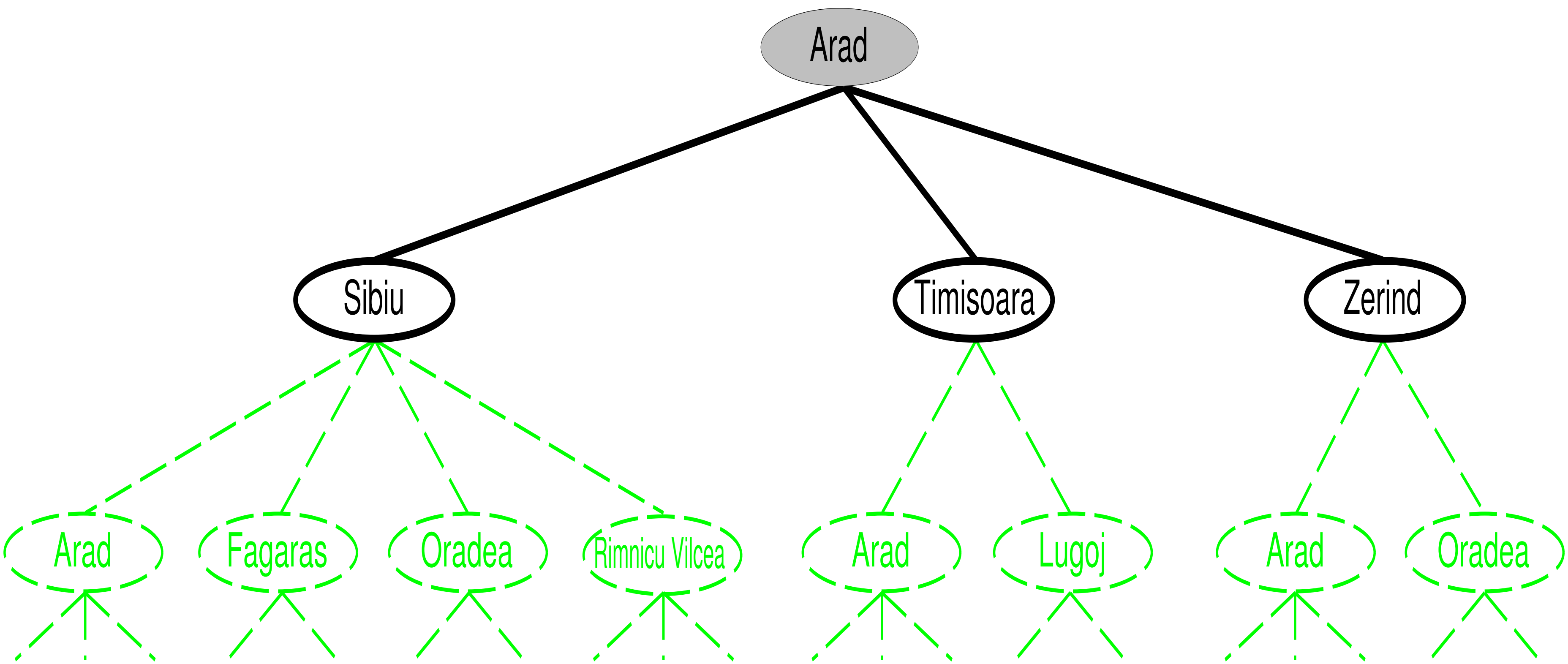
- **Offline**
- **Simulated exploration of state space in a search tree by generating successors of already-explored states**

function TREE-SEARCH(*problem, strategy*) **returns** a solution or failure
  initialize the search tree using the initial state of *problem*
  loop do
    if there are no candidates for expansion **then return** failure
    choose a leaf node for expansion according to *strategy*
    if the node contains a goal state **then**
      return the corresponding solution
    else
      expand the node and add the resulting nodes to the search tree
  end

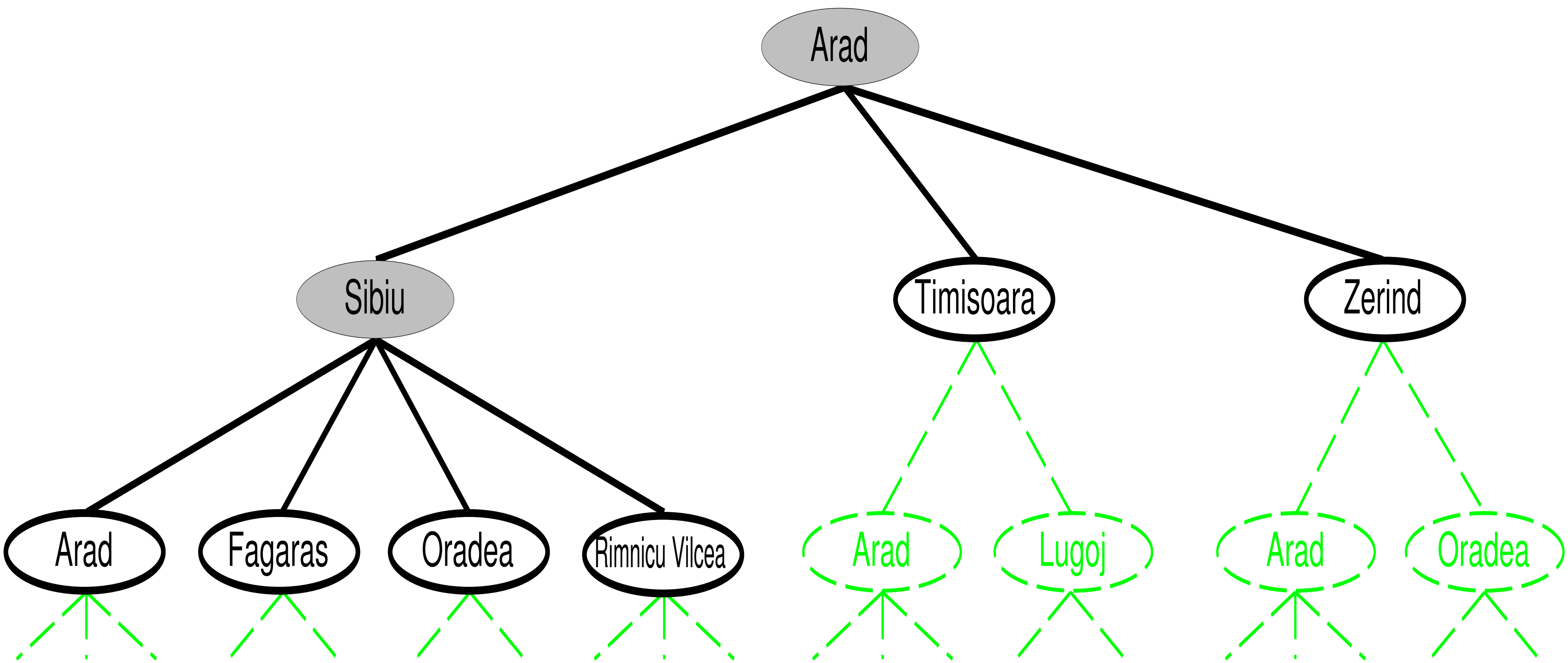
Tree search: Example



Tree search: Example



Tree search: Example



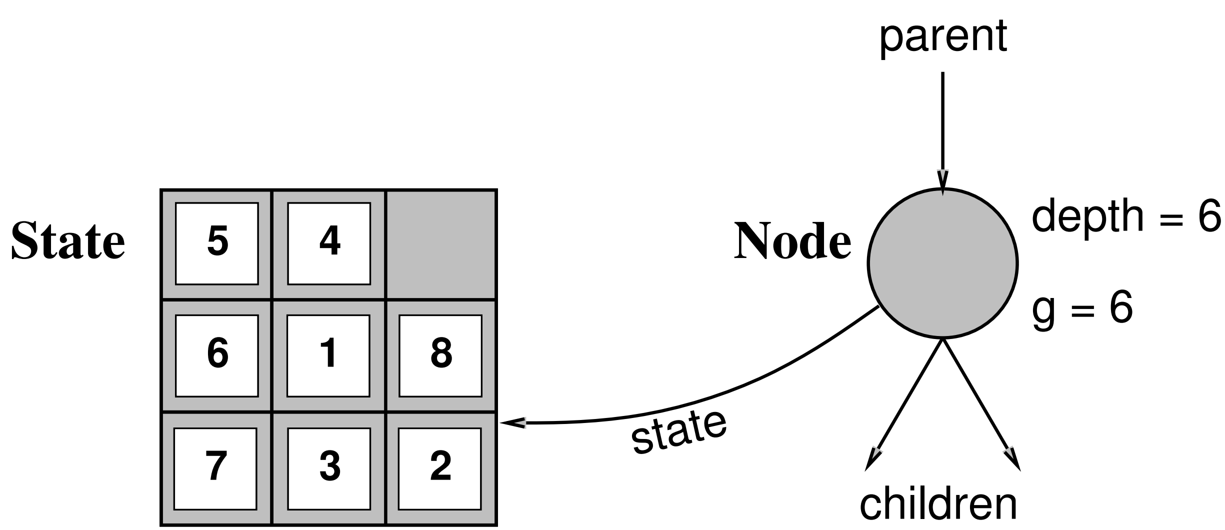
Implementation: States vs. nodes

State

A (representation of) a physical configuration

Node

A data structure constituting part of a search tree
(includes *parent*, *children*, *depth*, *path cost*, etc.)



Implementation of search algorithms

function TREE-SEARCH(*problem, fringe*) **returns** a solution or failure

fringe $\leftarrow$ INSERT(MAKE-NODE(INITIAL-STATE[*problem*]),*fringe*)
loop do
if *fringe* is empty **then return** failure
node $\leftarrow$ REMOVE-FIRST(*fringe*)
if GOAL-TEST[*problem*] applied to STATE(*node*) succeeds **then**
return *node*
else
fringe $\leftarrow$ INSERT-ALL(EXPAND(*node*, *problem*), *fringe*)
end

fringe **queue of nodes not yet considered**
State **gives the state that is represented by *node***
Expand **creates new nodes by applying possible actions to *node***

Search strategies

Strategy

Defines the order of node expansion

Important properties of strategies

completeness	does it always find a solution if one exists?
time complexity	number of nodes generated/expanded
space complexity	maximum number of nodes in memory
optimality	does it always find a least-cost solution?

Time and space complexity measured in terms of

b	maximum branching factor of the search tree
d	depth of a solution with minimal distance to root
m	maximum depth of the state space (may be ∞)

Uninformed search strategies

Uninformed search

Use only the information available in the problem definition

Frequently used strategies

- **Breadth-first search**
- **Uniform-cost search**
- **Depth-first search**
- **Depth-limited search**
- **Iterative deepening search**

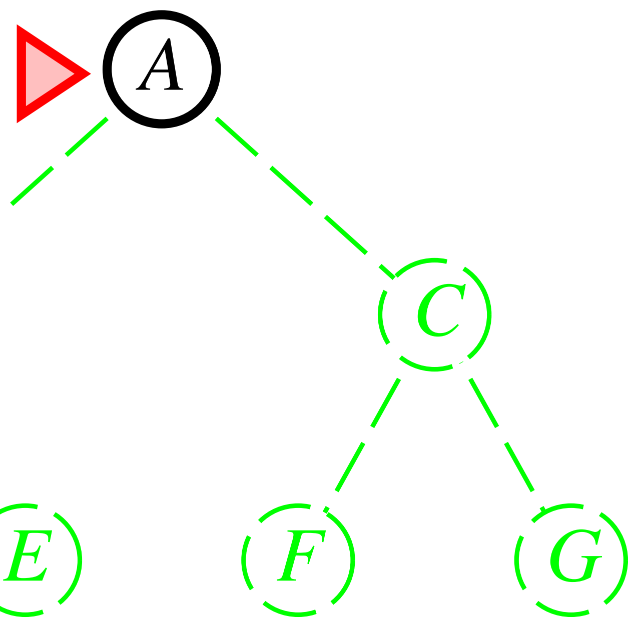
Breadth-first search

Idea

Expand shallowest unexpanded node

Implementation

***fringe* is a FIFO queue, i.e. successors go in at the end of the queue**



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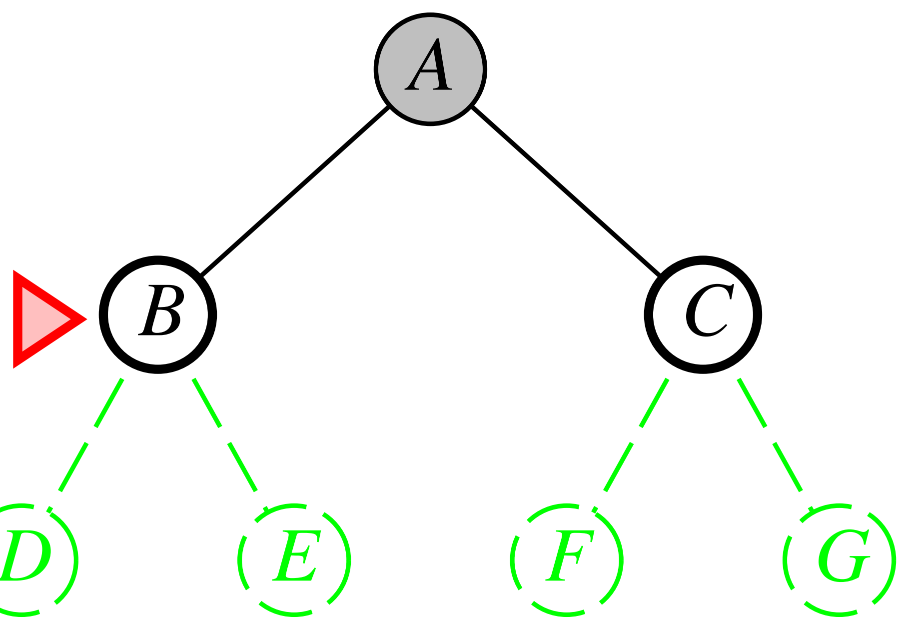

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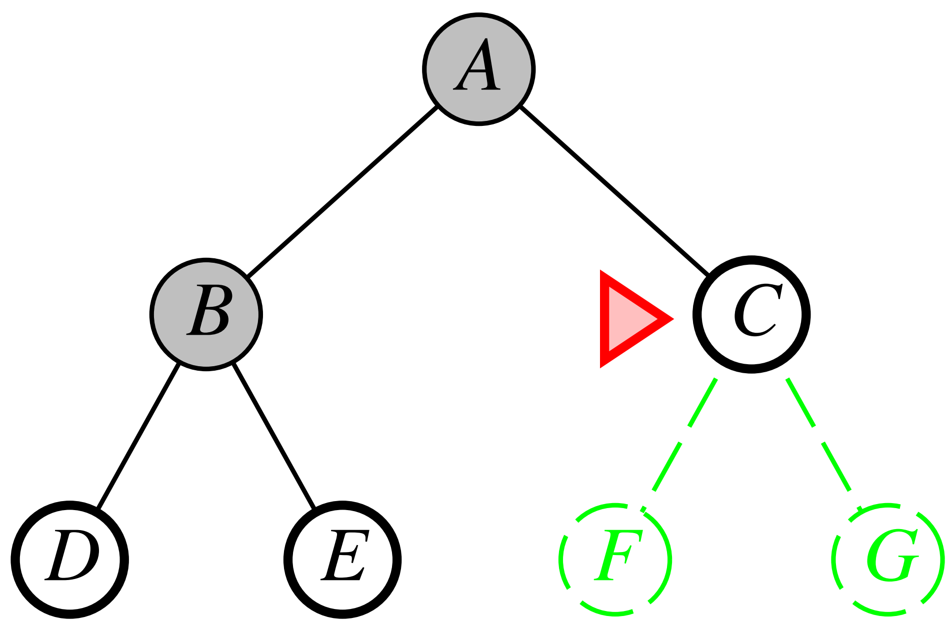

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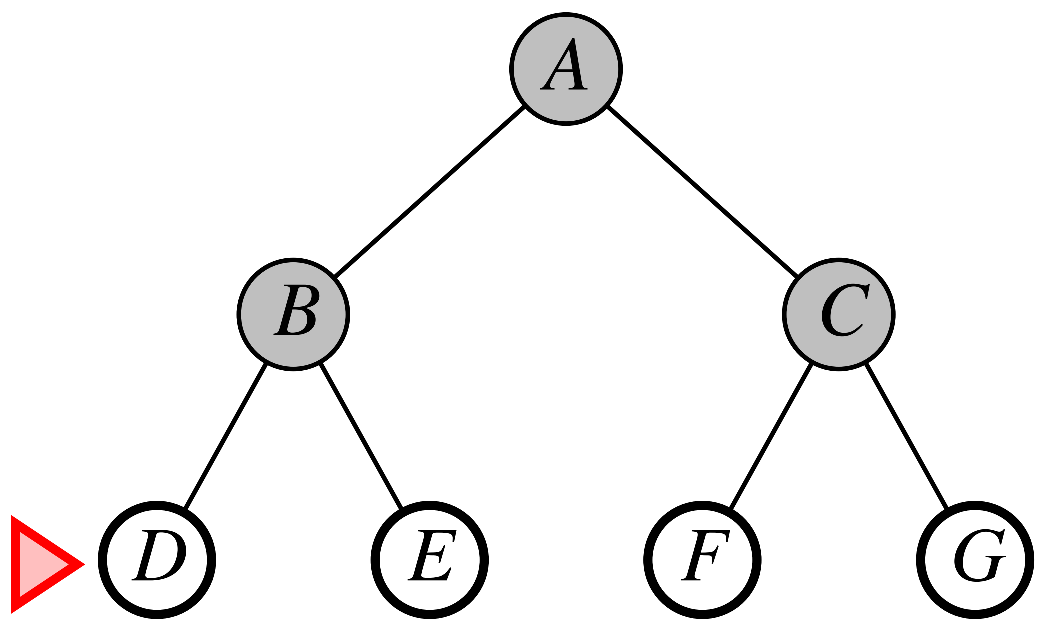
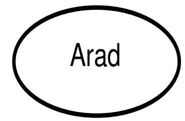


Breadth-first search: Example Romania



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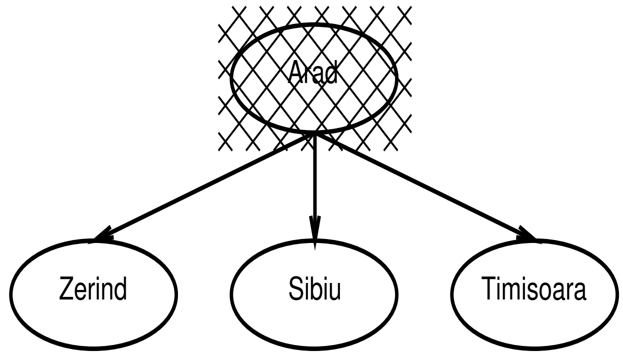

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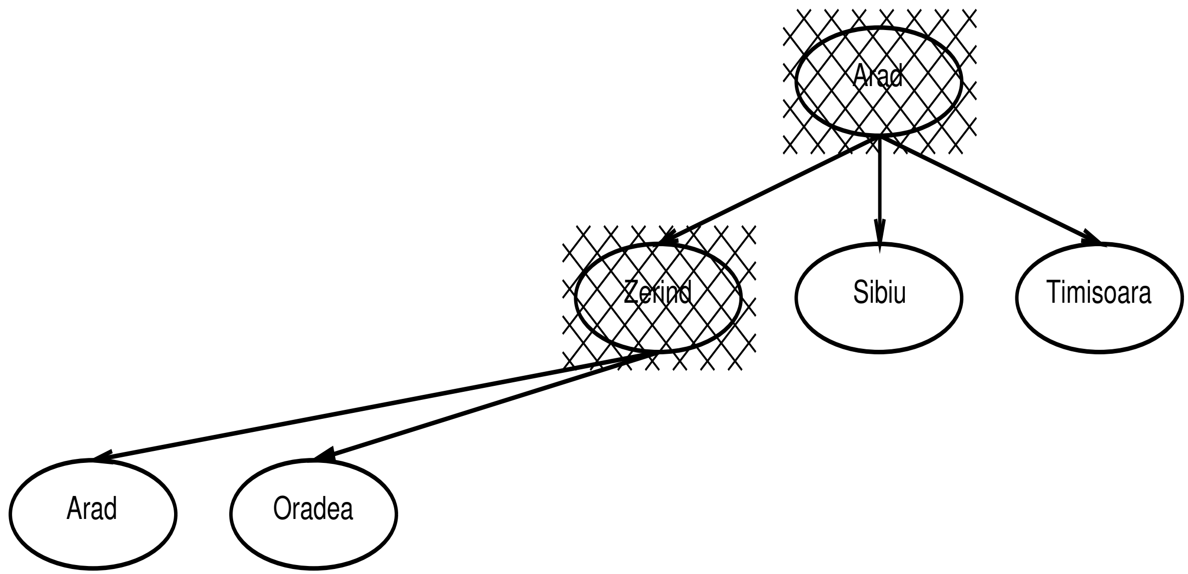

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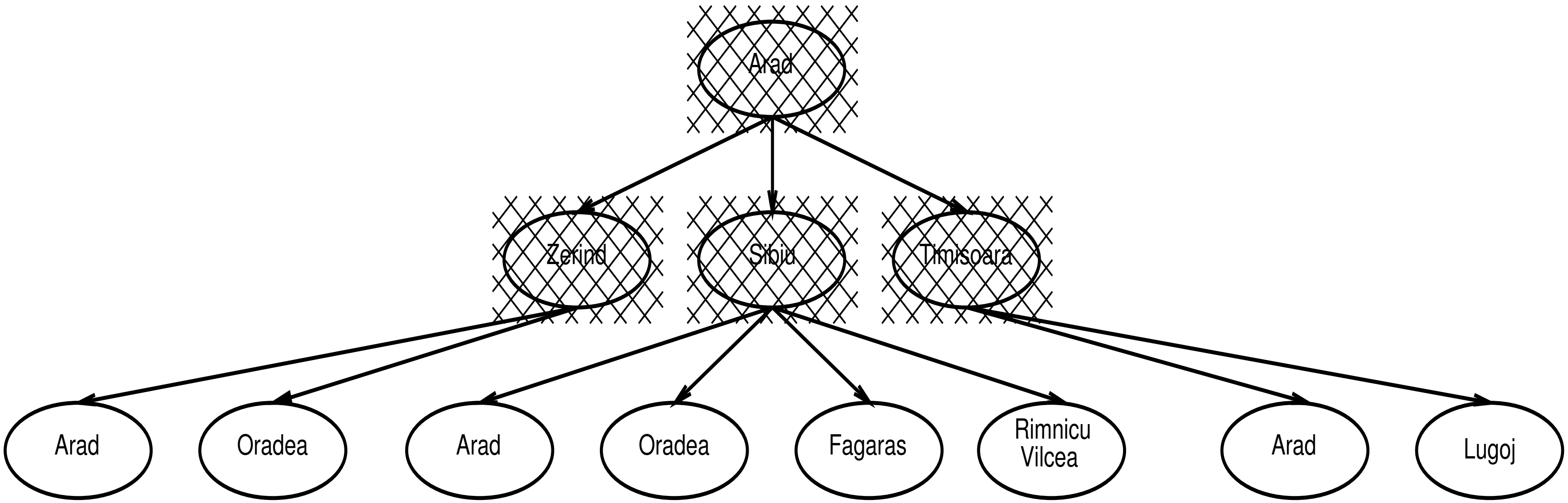

Breadth-first search: Properties

Complete

Time

Space

Optimal

Breadth-first search: Properties

Complete **Yes (if b is finite)**

Time

Space

Optimal

Breadth-first search: Properties

Complete **Yes (if b is finite)**

Time $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) \in O(b^{d+1})$

i.e. exponential in d

Space

Optimal

Breadth-first search: Properties

Complete Yes (if b is finite)

Time $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) \in O(b^{d+1})$

i.e. exponential in d

Space $O(b^{d+1})$

keeps every node in memory

Optimal

Breadth-first search: Properties

Complete Yes (if b is finite)

Time $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) \in O(b^{d+1})$

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Space $O(b^{d+1})$

keeps every node in memory

Optimal Yes (if cost = 1 per step), not optimal in general

Breadth-first search: Properties

Complete Yes (if b is finite)

Time $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) \in O(b^{d+1})$

i.e. exponential in d

Space $O(b^{d+1})$

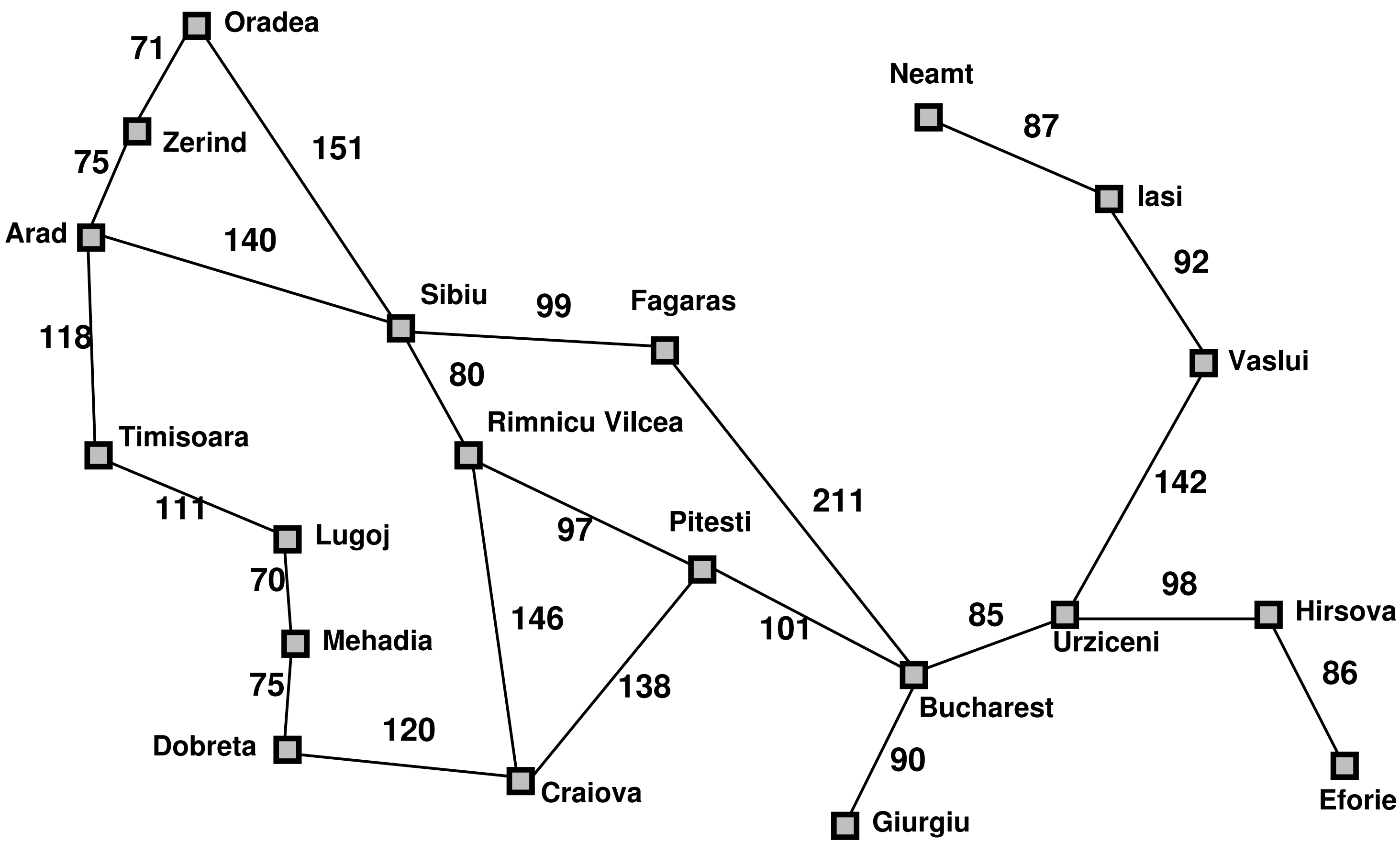
keeps every node in memory

Optimal Yes (if cost = 1 per step), not optimal in general

Disadvantage

Space is the big problem
(can easily generate nodes at 5MB/sec so 24hrs = 430GB)

Romania with step costs in km



Straight−line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Uniform-cost search

Idea

Expand least-cost unexpanded node
(costs added up over paths from root to leafs)

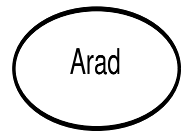
Implementation

fringe is queue ordered by increasing path cost

Note

Equivalent to depth-first search if all step costs are equal

Uniform-cost search



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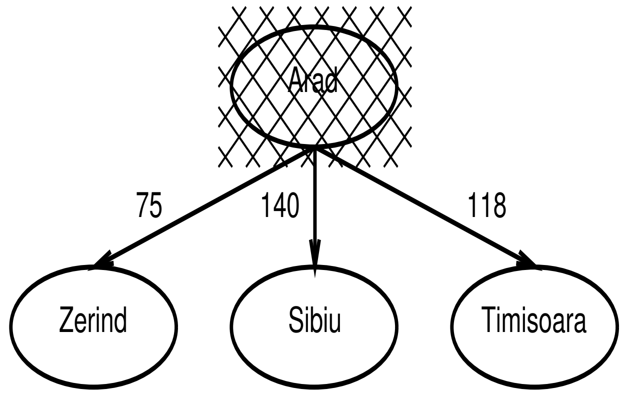

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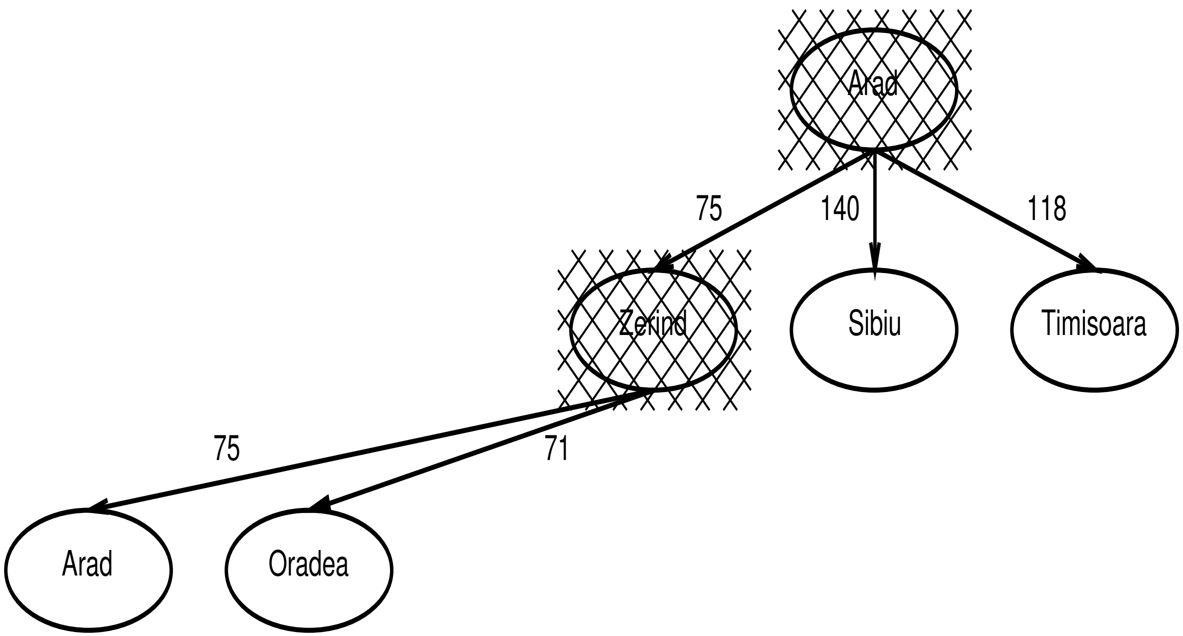

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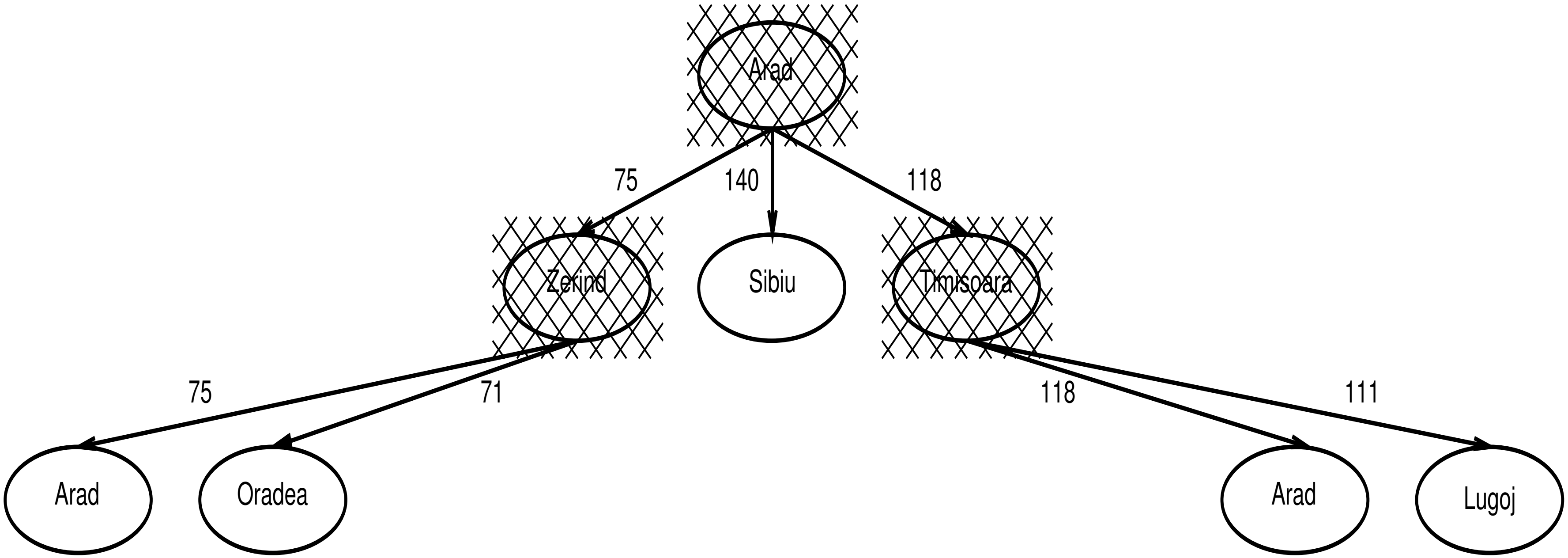


Uniform-cost search: Properties

Complete

Time

Space

Optimal

Uniform-cost search: Properties

Complete	**Yes (if step costs positive)**
Time	
Space	
Optimal	

Uniform-cost search: Properties

Complete	Yes (if step costs positive)
Time	# of nodes with past-cost less than that of optimal solution
Space	
Optimal	

Uniform-cost search: Properties

Complete	Yes (if step costs positive)
Time	# of nodes with past-cost less than that of optimal solution
Space	# of nodes with past-cost less than that of optimal solution
Optimal	

Uniform-cost search: Properties

Complete	Yes (if step costs positive)
Time	# of nodes with past-cost less than that of optimal solution
Space	# of nodes with past-cost less than that of optimal solution
Optimal	Yes

Depth-first search

Idea

Expand deepest unexpanded node

Implementation

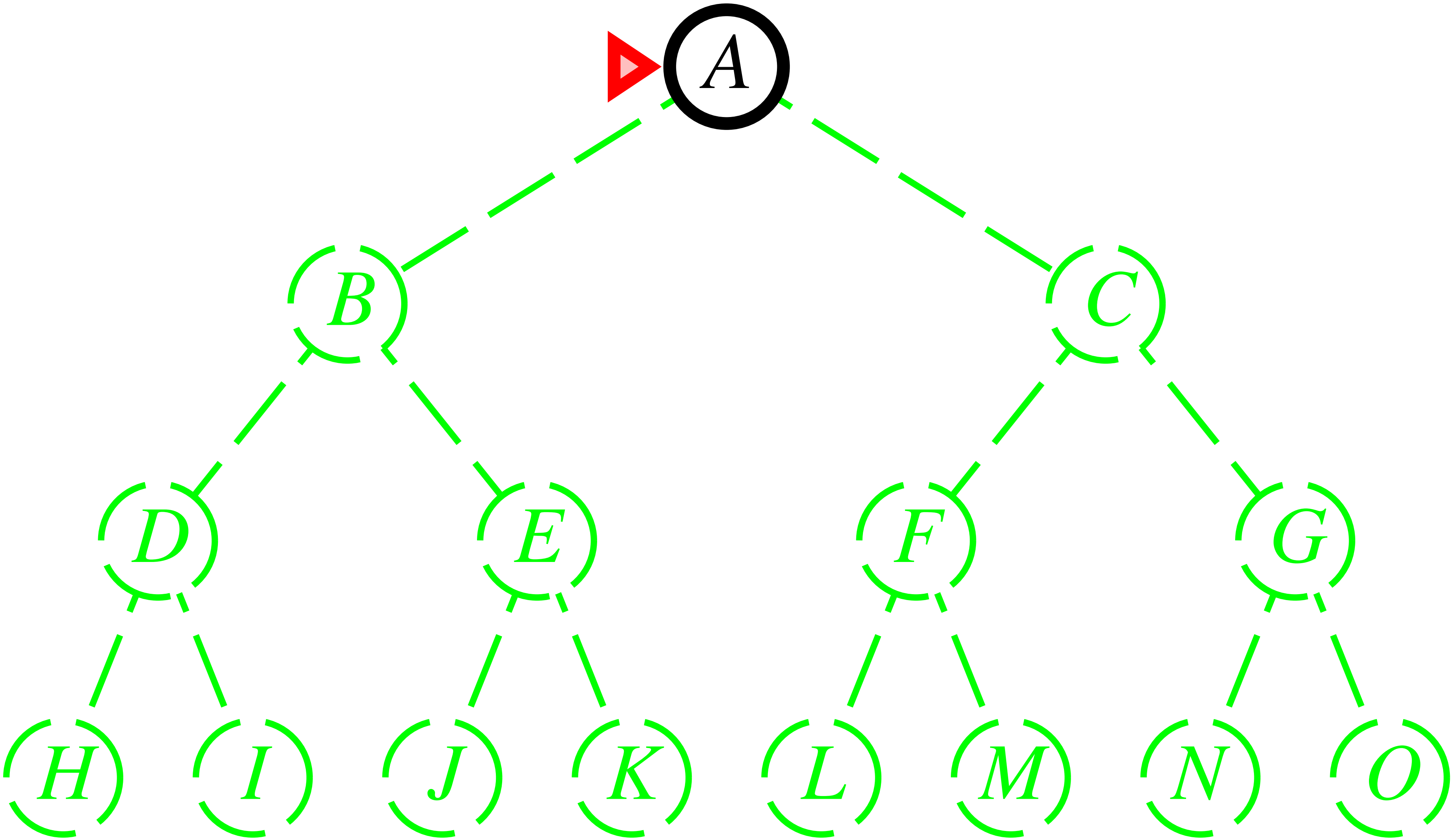
fringe **is a LIFO queue (a stack), i.e. successors go in at front of queue**

Note

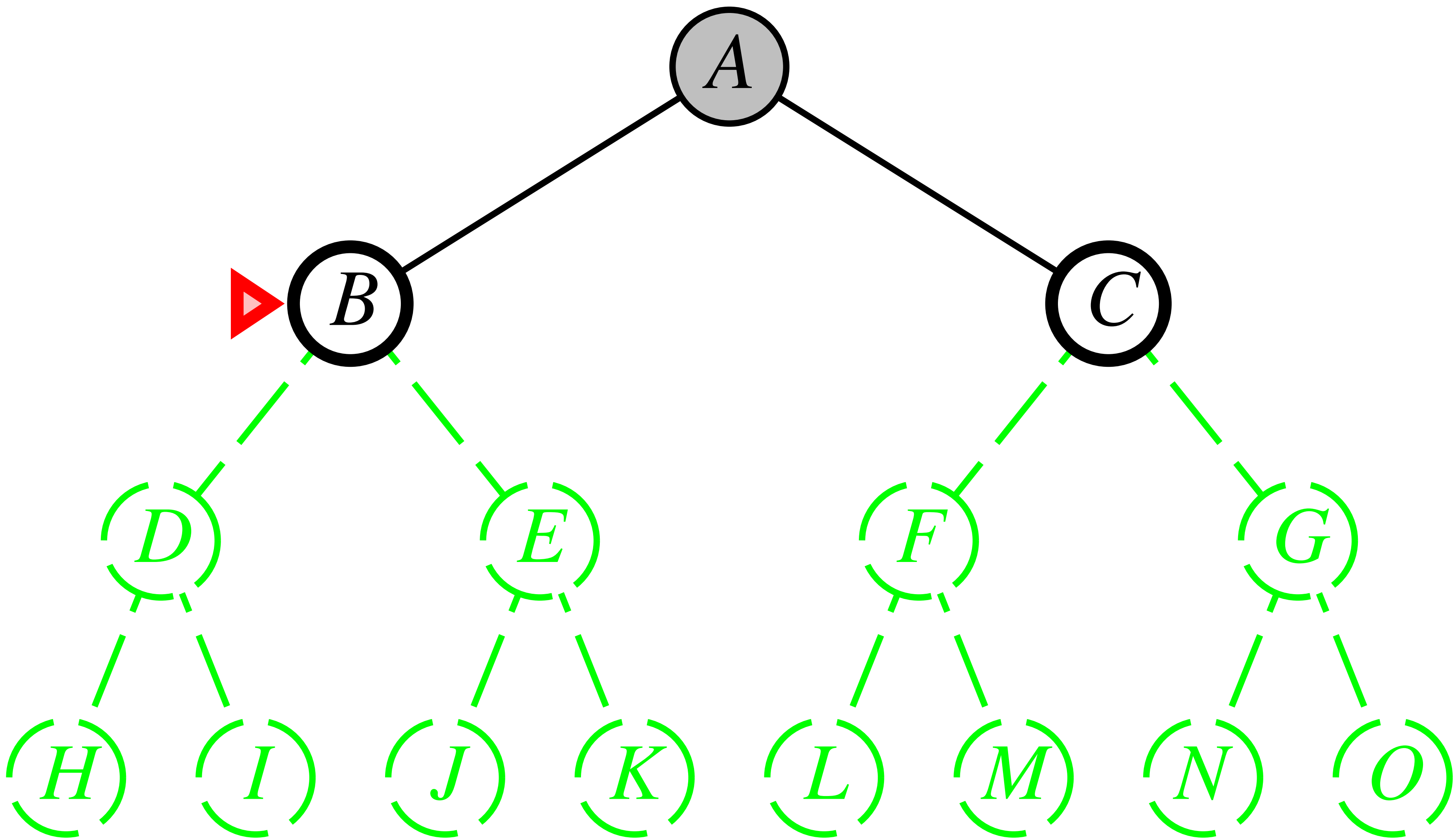
Depth-first search can perform infinite cyclic excursions

Need a finite, non-cyclic search space (or repeated-state checking)

Depth-first search



Depth-first search



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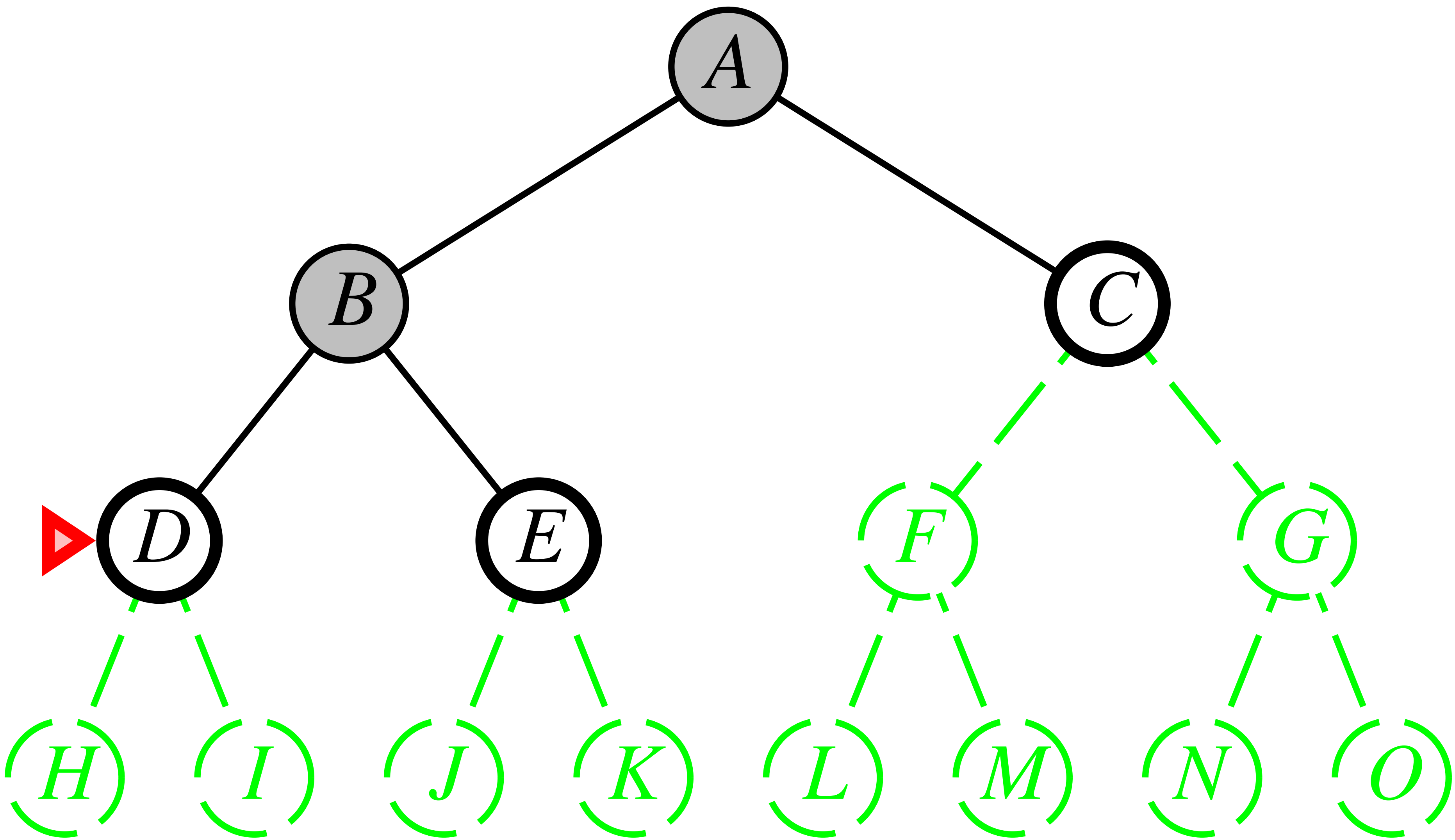
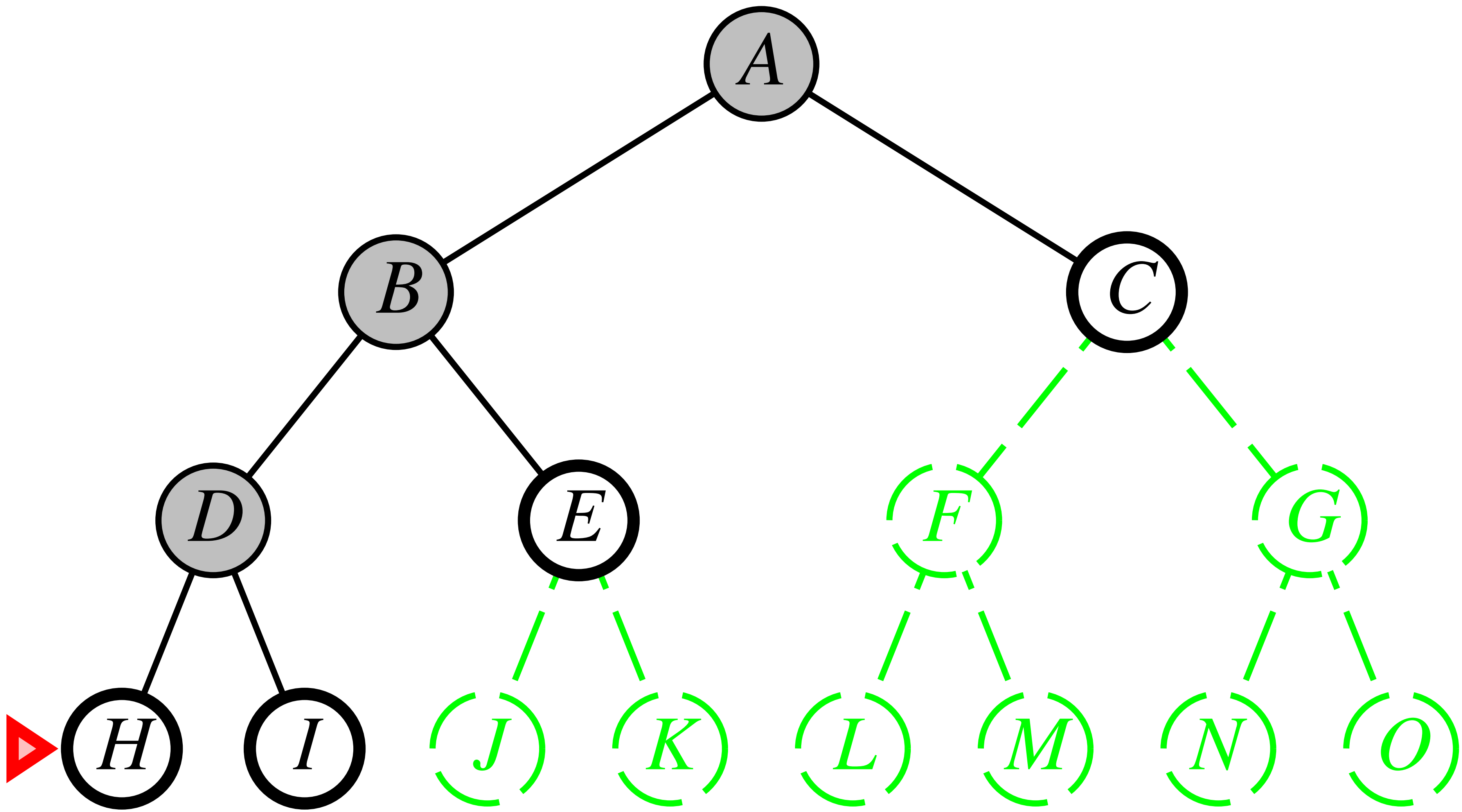
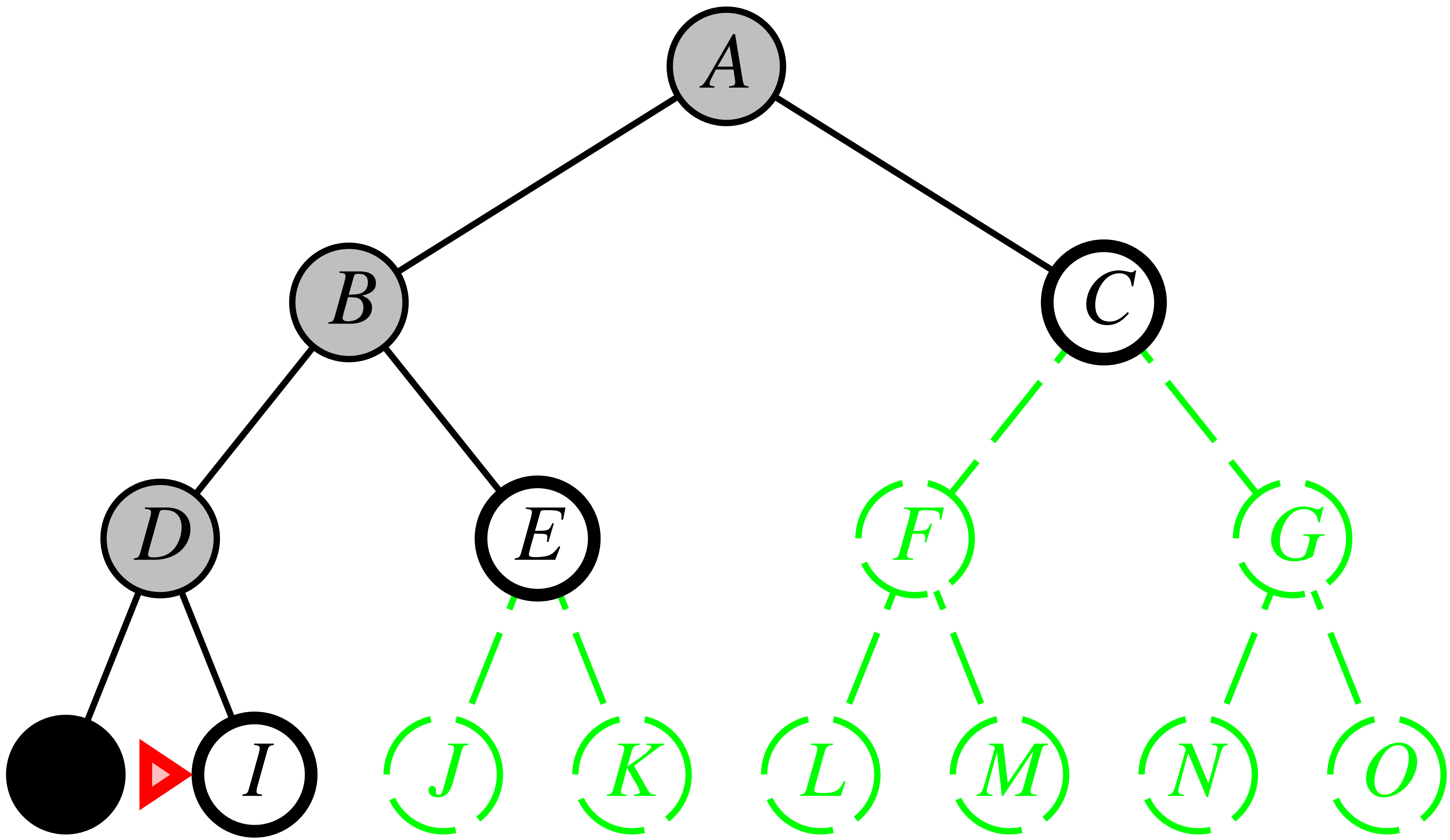


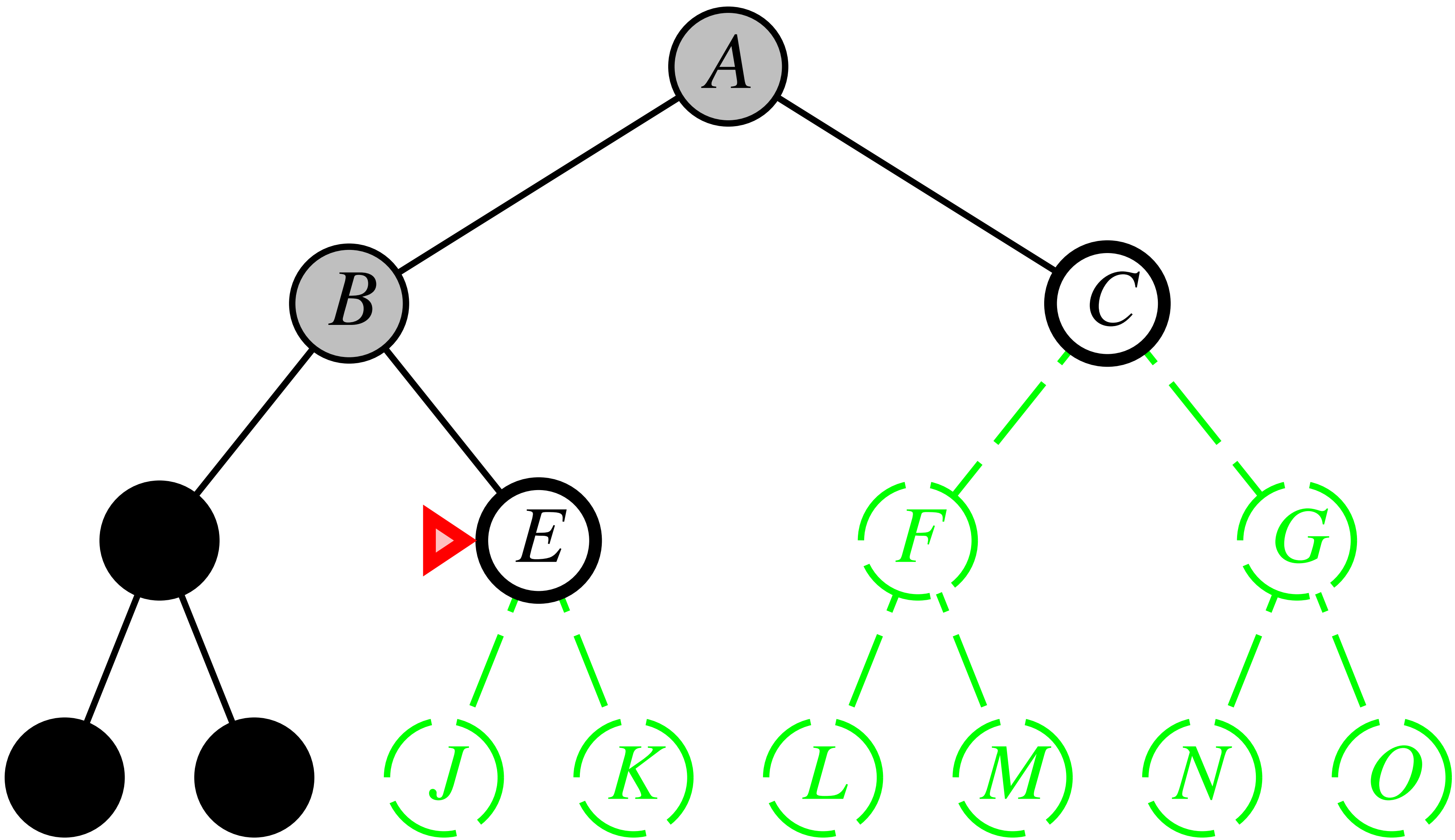
Depth-first search



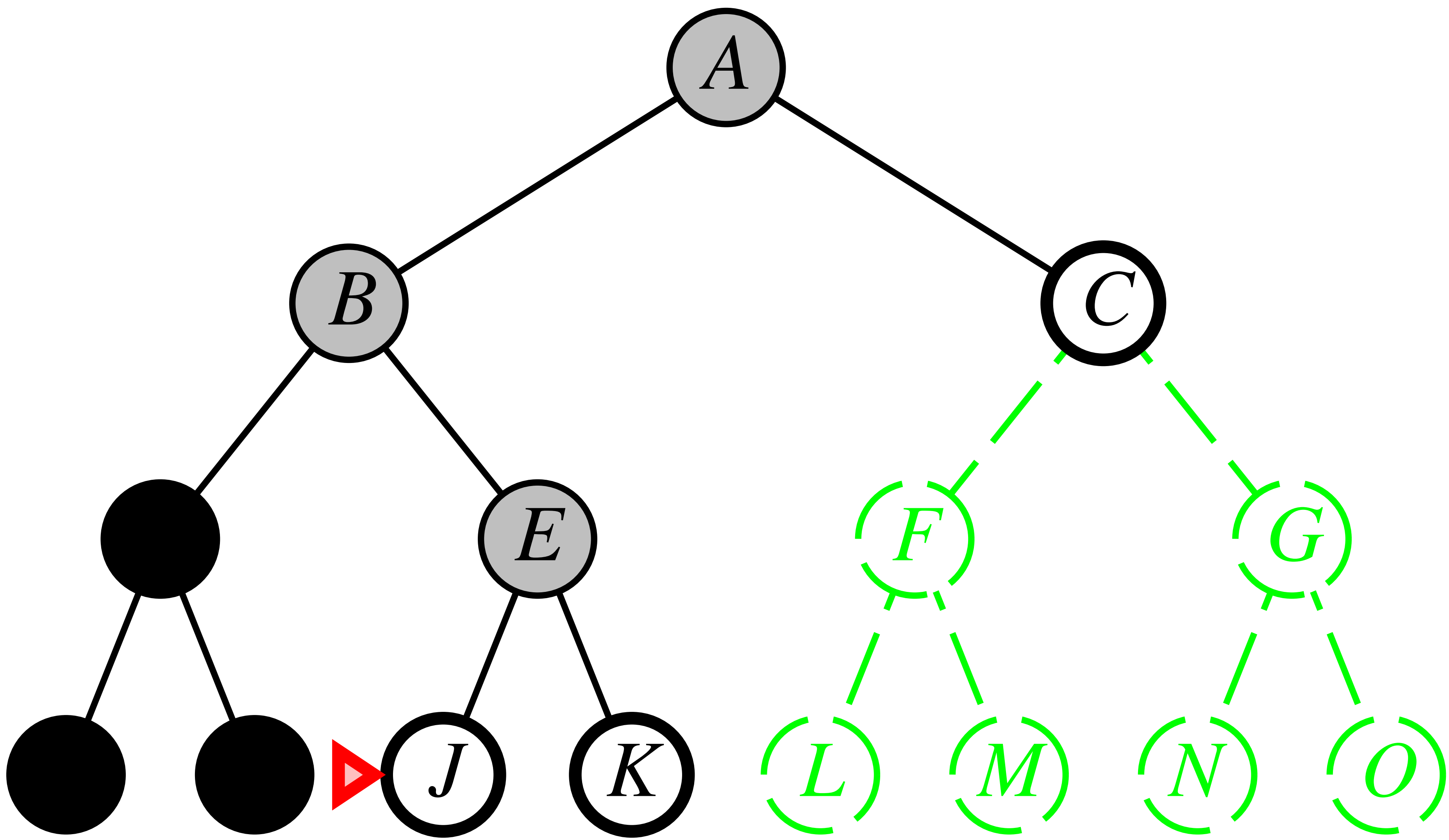
Depth-first search



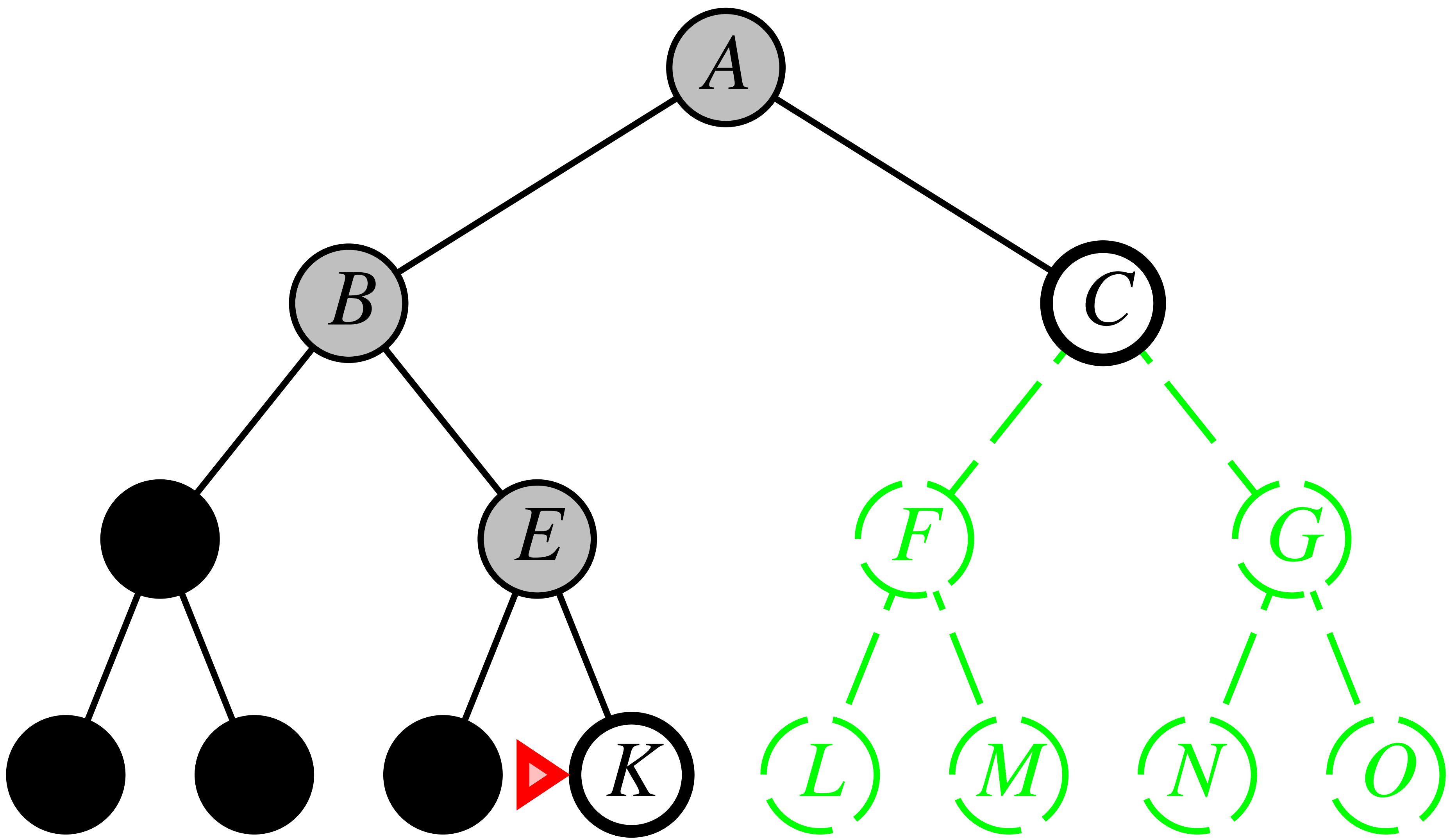
Depth-first search



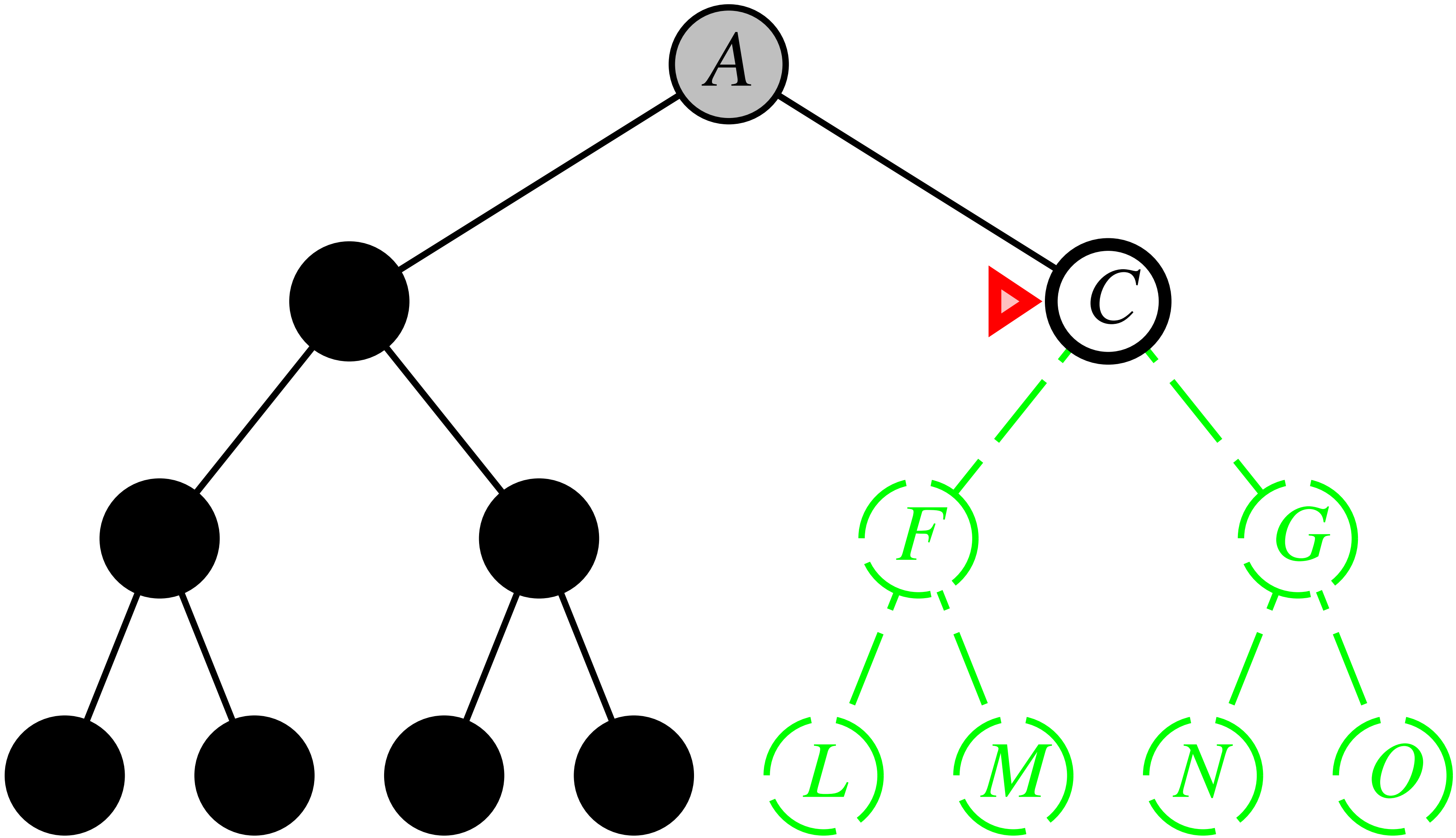
Depth-first search



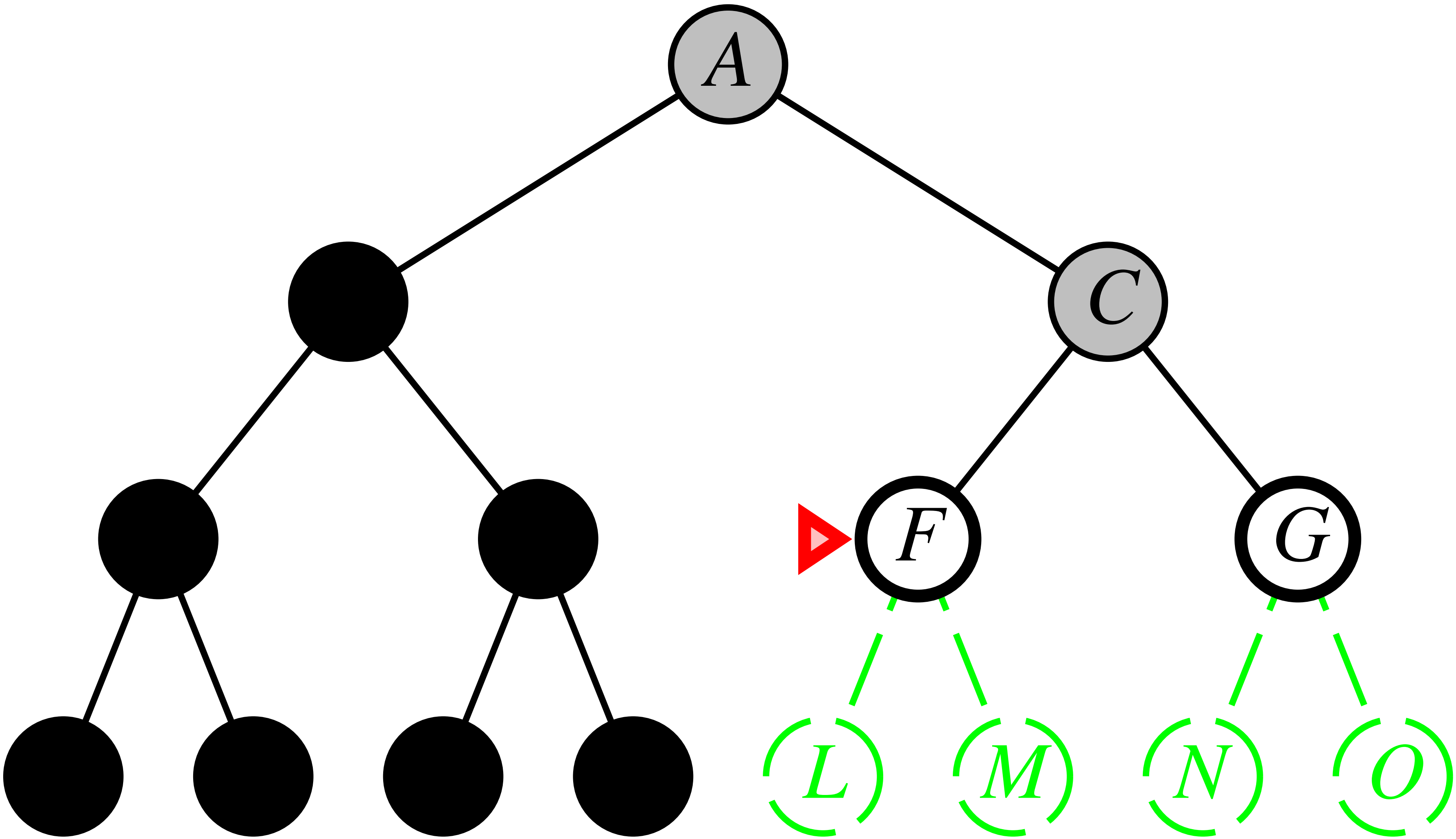
Depth-first search



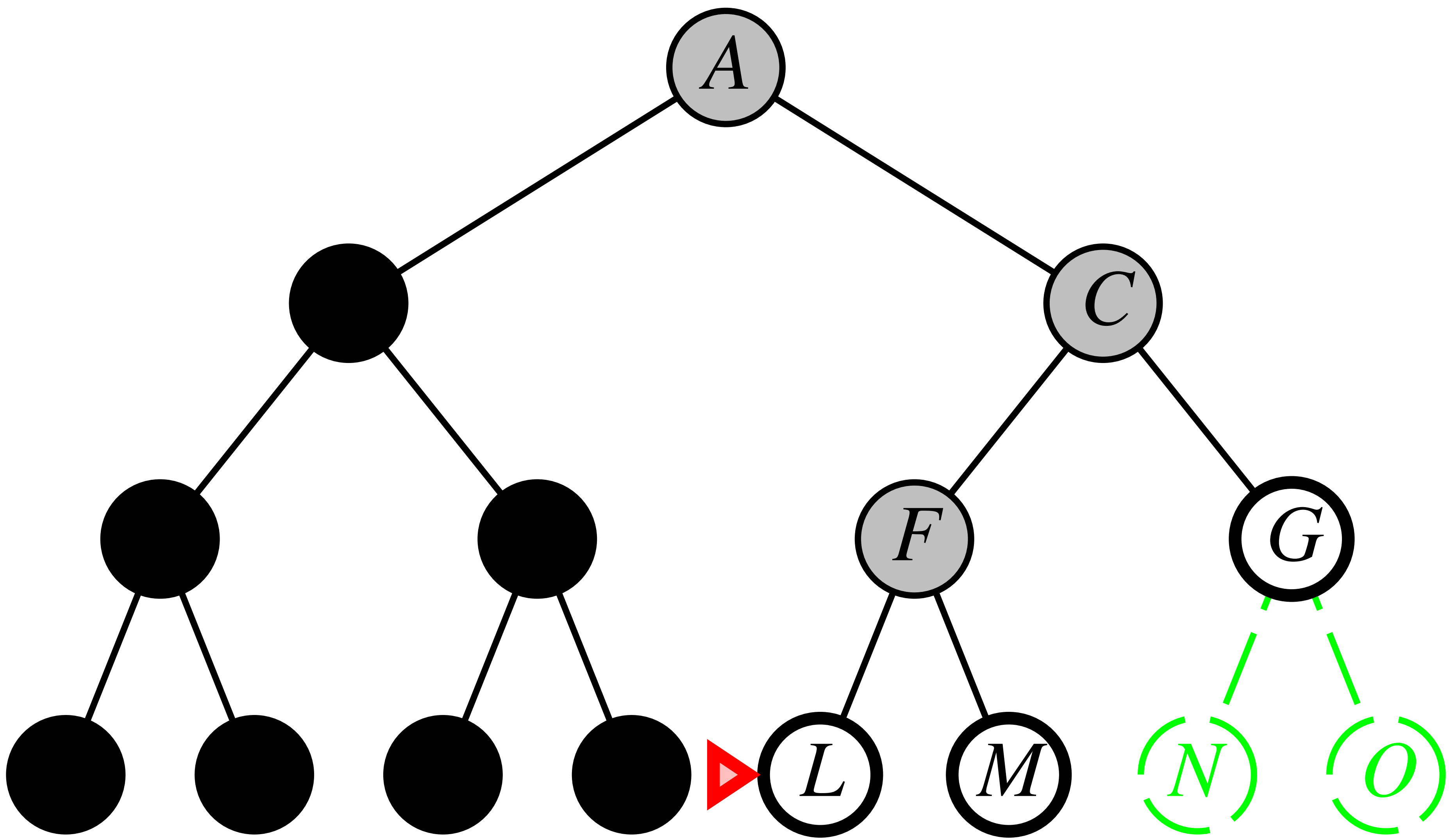
Depth-first search



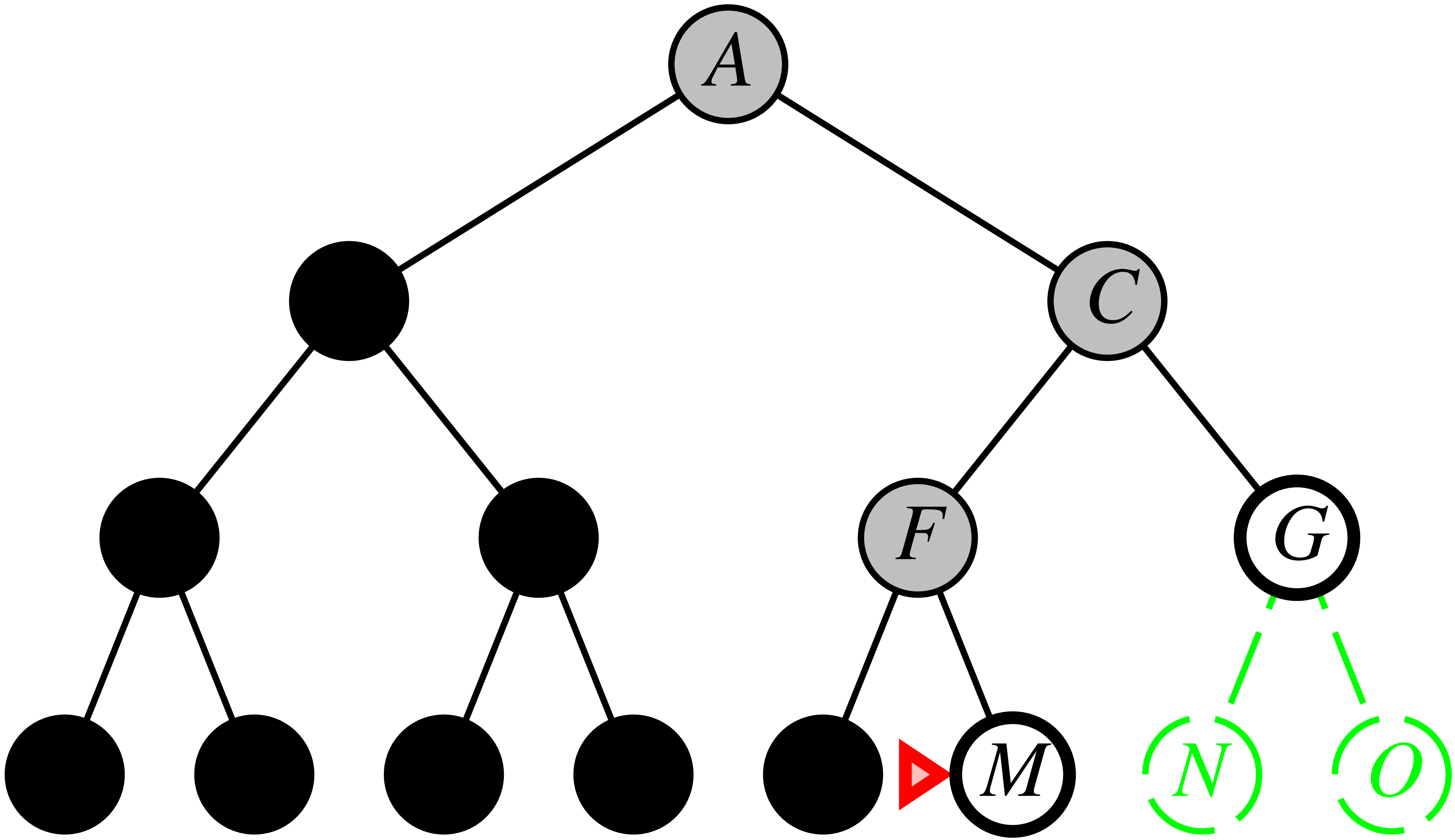
Depth-first search



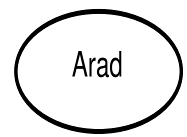
Depth-first search



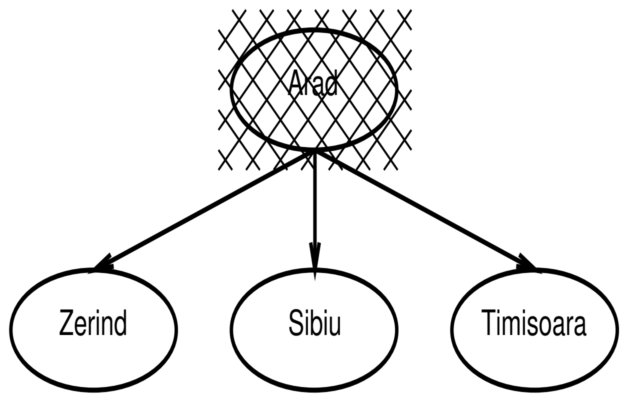
Depth-first search



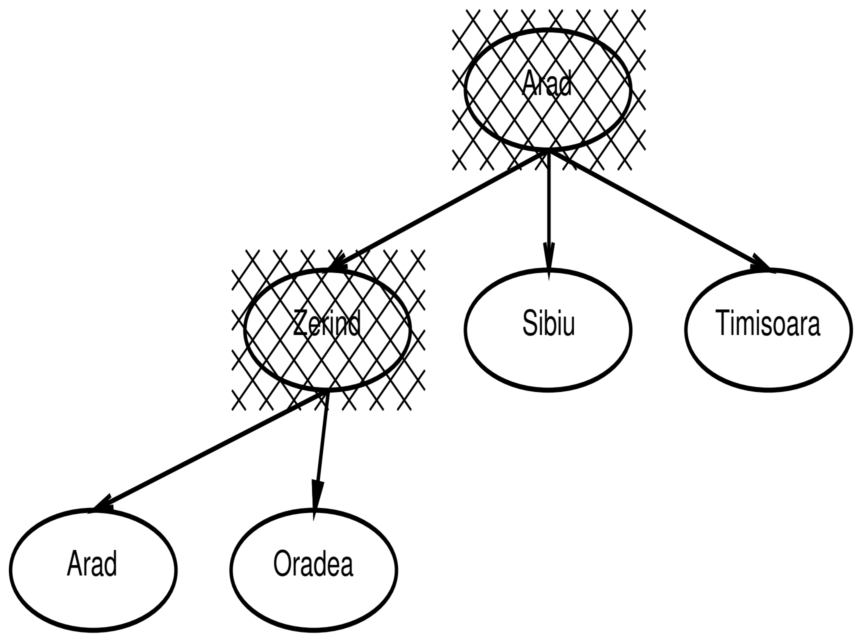
Depth-first search: Example Romania



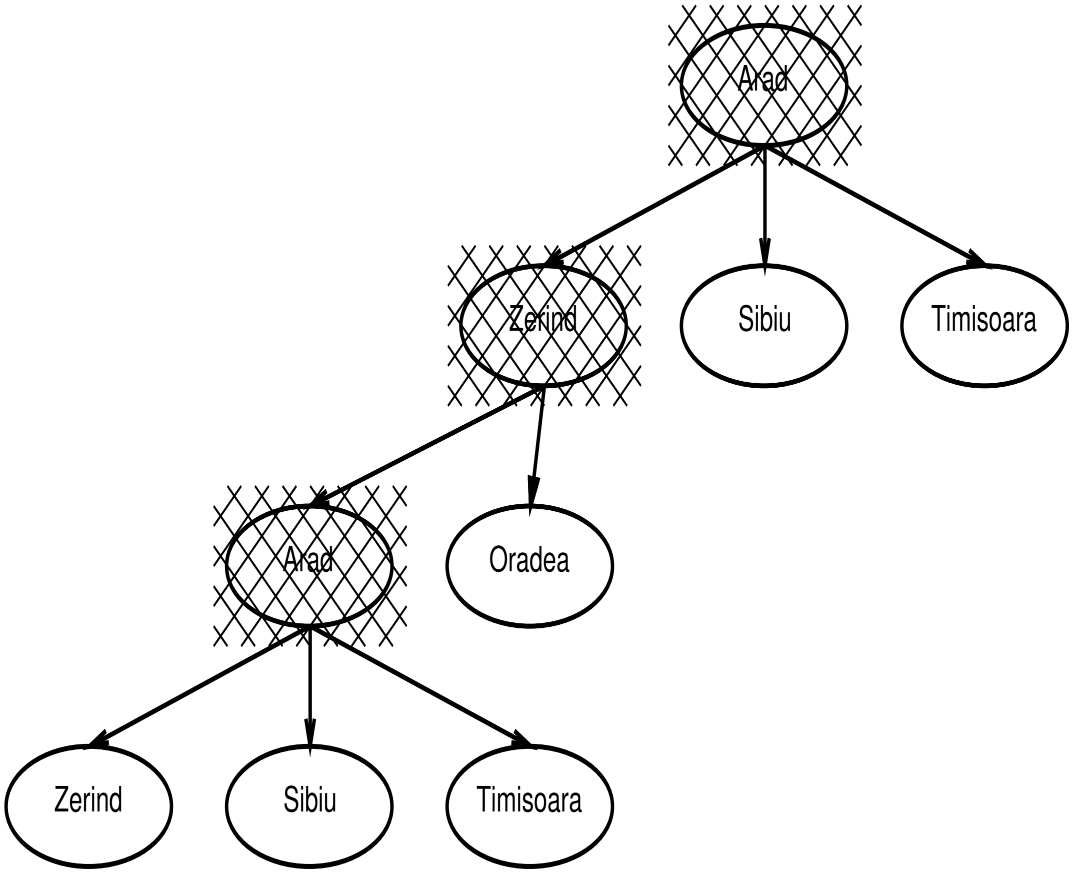
Depth-first search: Example Romania



Depth-first search: Example Romania



Depth-first search: Example Romania



Depth-first search: Properties

Complete

Time

Space

Optimal

Depth-first search: Properties

Complete	**Yes:**	**if state space finite**
	No:	**if state contains infinite paths or loops**
Time		
Space		
Optimal		

Depth-first search: Properties

Complete	**Yes: if state space finite**
	No: if state contains infinite paths or loops
Time	$O(b^m)$
Space	
Optimal	

Depth-first search: Properties

Complete	**Yes: if state space finite**
	No: if state contains infinite paths or loops
Time	$O(b^m)$
Space	$O(bm)$ **(i.e. linear space)**
Optimal	

Depth-first search: Properties

Complete	**Yes:** if state space finite
	No: if state contains infinite paths or loops
Time	$O(b^m)$
Space	$O(bm)$ **(i.e. linear space)**
Optimal	**No**

Depth-first search: Properties

Complete Yes: if state space finite

No: if state contains infinite paths or loops

Time $O(b^m)$

Space $O(bm)$ (i.e. linear space)

Optimal No

Disadvantage

Time terrible if m much larger than d

Advantage

Time may be much less than breadth-first search if solutions are dense

Iterative deepening search

Depth-limited search

Depth-first search with depth limit

Iterative deepening search

Depth-limited search

Depth-first search with depth limit

Iterative deepening search

Depth-limit search with ever increasing limits

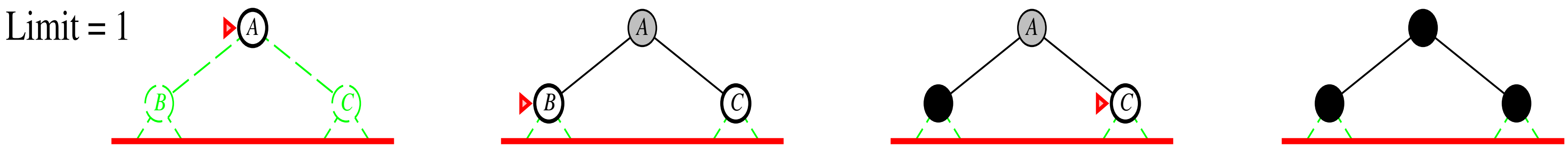
function ITERATIVE-DEEPENING-SEARCH(*problem*) **returns** a solution or failure
  inputs: *problem*   /* a problem */

  for *depth* $\leftarrow$ 0 **to** ∞ **do**
    result $\leftarrow$ DEPTH-LIMITED-SEARCH(*problem*, *depth*)
    if *result* $\neq$ cutoff **then return** *result*
  end

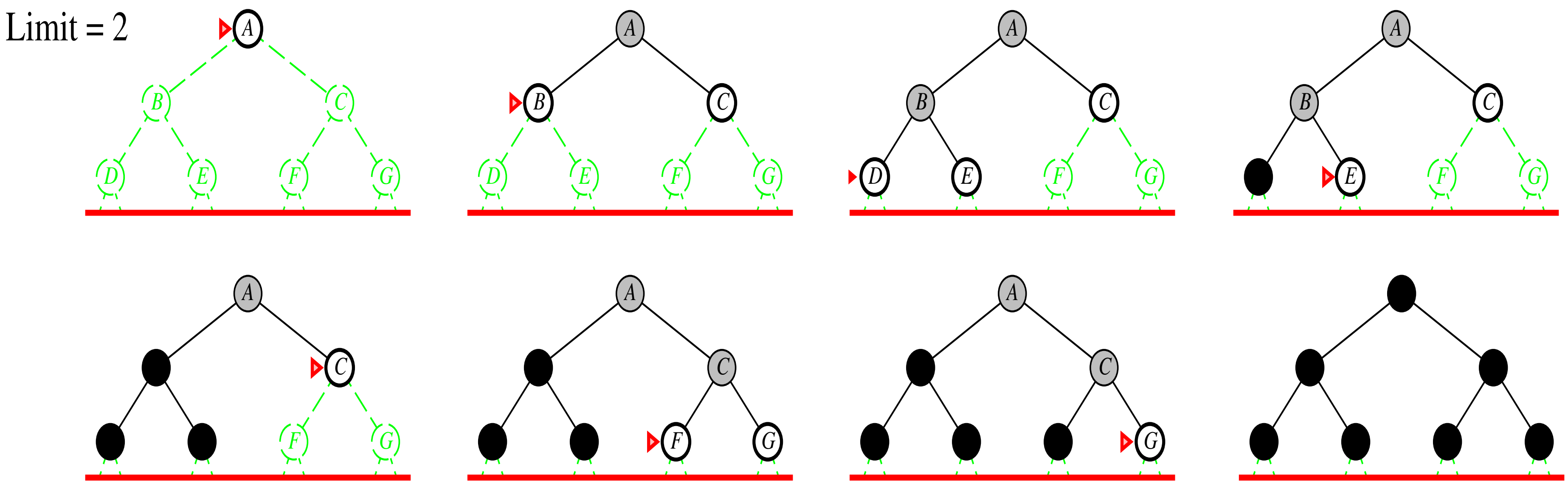
Iterative deepening search with depth limit 0

Limit = 0

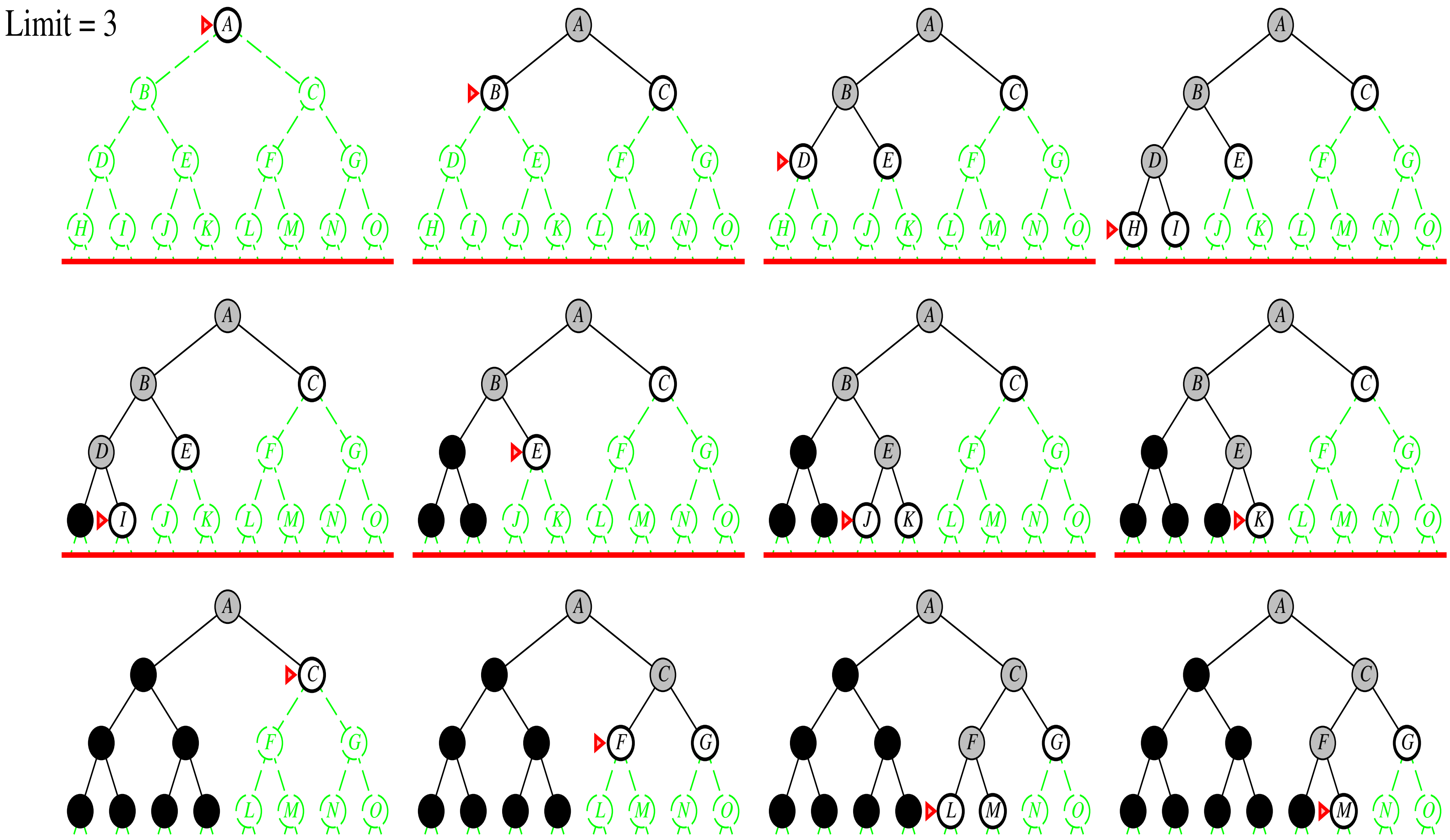
Iterative deepening search with depth limit 1



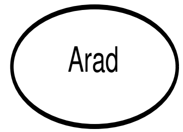
Iterative deepening search with depth limit 2



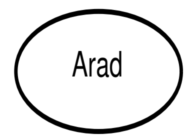
Iterative deepening search with depth limit 3



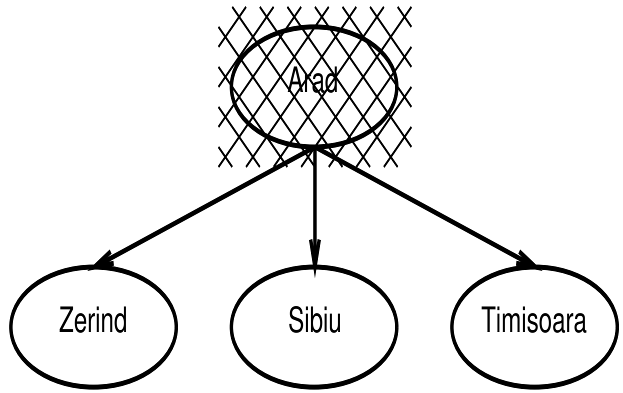
Iterative deepening search: Example Romania with $l = 0$



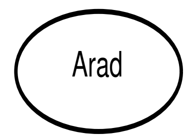
Iterative deepening search: Example Romania with $l = 1$



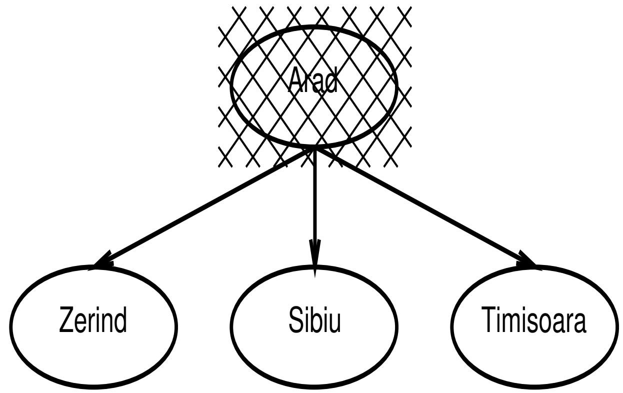
Iterative deepening search: Example Romania with $l = 1$



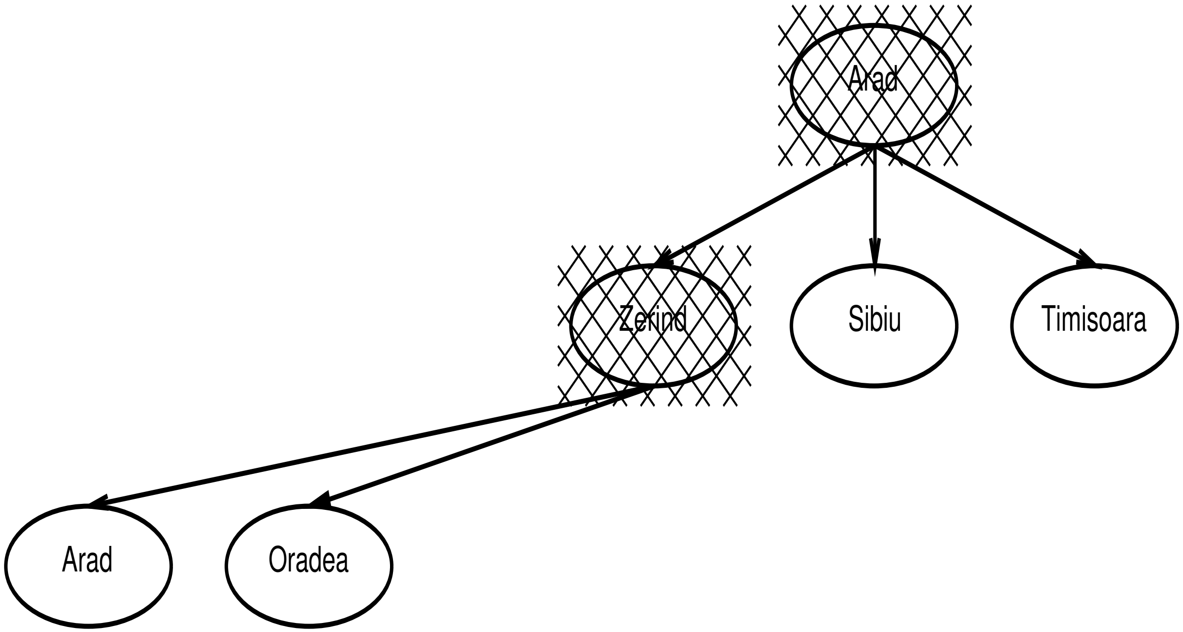
Iterative deepening search: Example Romania with $l = 2$



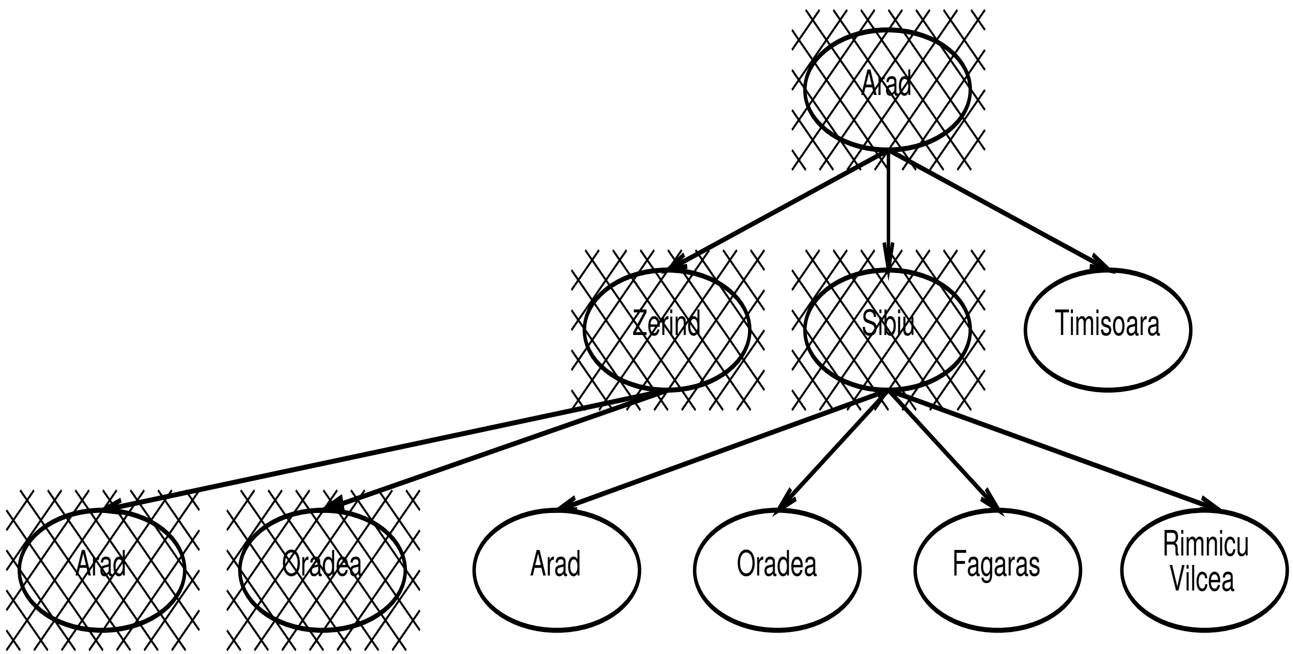
Iterative deepening search: Example Romania with $l = 2$



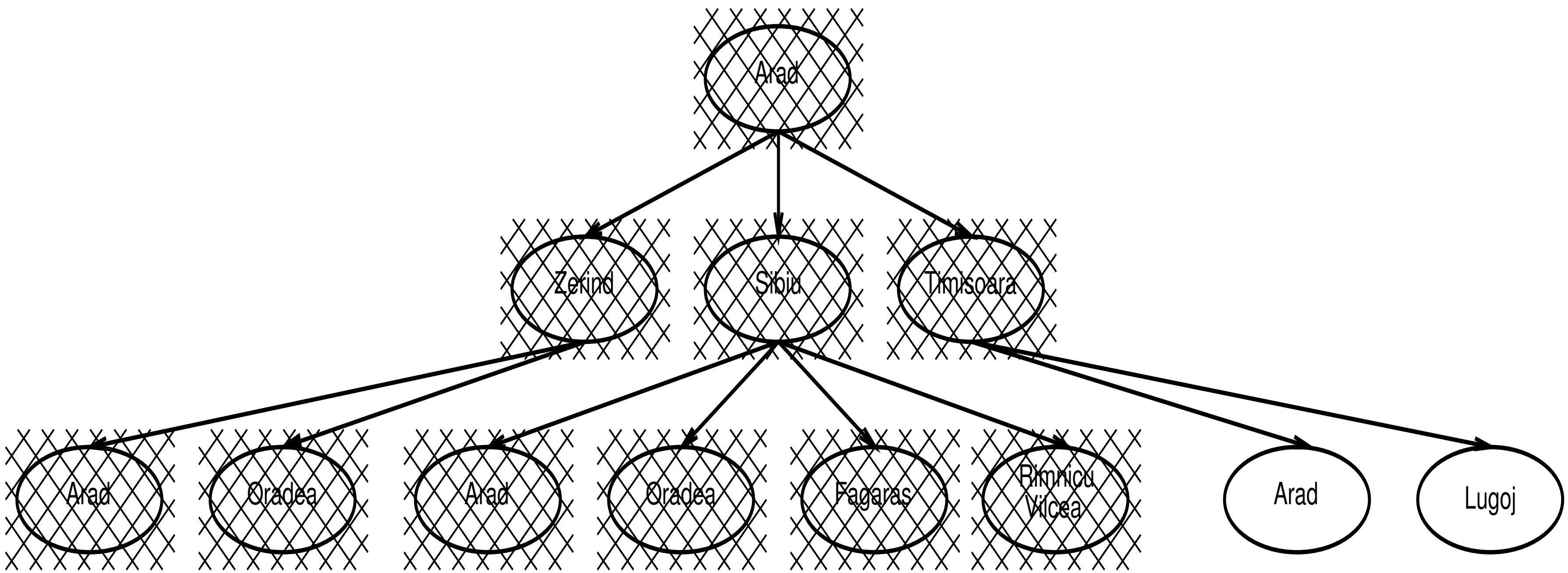
Iterative deepening search: Example Romania with $l = 2$



Iterative deepening search: Example Romania with $l = 2$



Iterative deepening search: Example Romania with $l = 2$



Iterative deepening search: Properties

Complete

Time

Space

Optimal

Iterative deepening search: Properties

Complete Yes

Time

Space

Optimal

Iterative deepening search: Properties

Complete **Yes**

Time $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d \in O(b^{d+1})$

Space

Optimal

Iterative deepening search: Properties

Complete **Yes**

Time $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d \in O(b^{d+1})$

Space $O(bd)$

Optimal

Iterative deepening search: Properties

Complete **Yes**

Time $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d \in O(b^{d+1})$

Space $O(bd)$

Optimal **Yes (if step cost = 1)**

Iterative deepening search: Properties

Complete **Yes**

Time $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d \in O(b^{d+1})$

Space $O(bd)$

Optimal **Yes (if step cost = 1)**

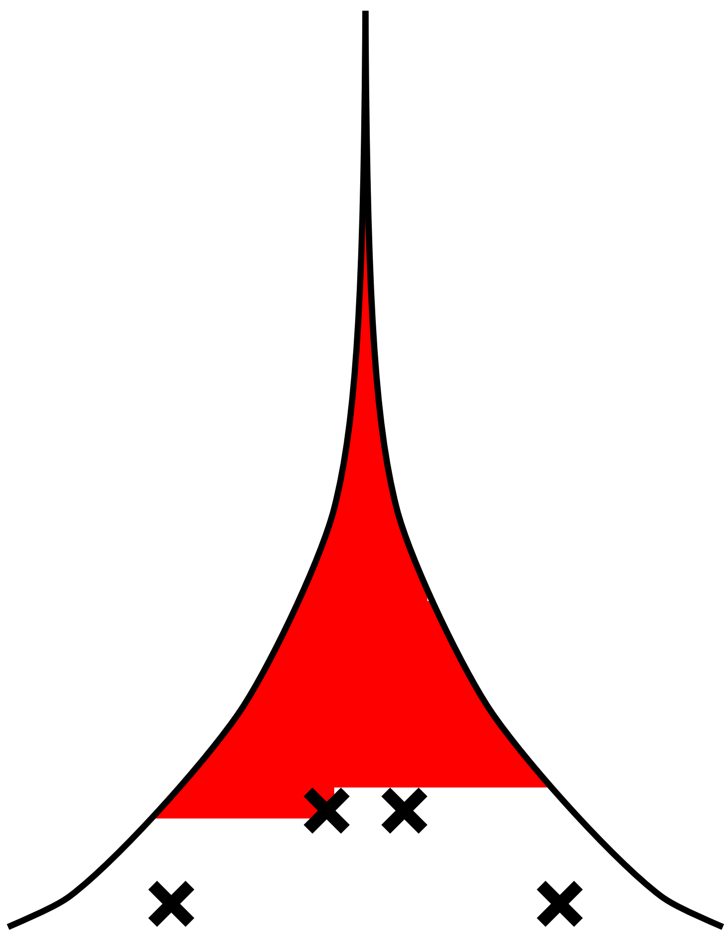
(Depth-First) Iterative-Deepening Search often used in practice for search spaces of large, infinite, or unknown depth.

Comparison

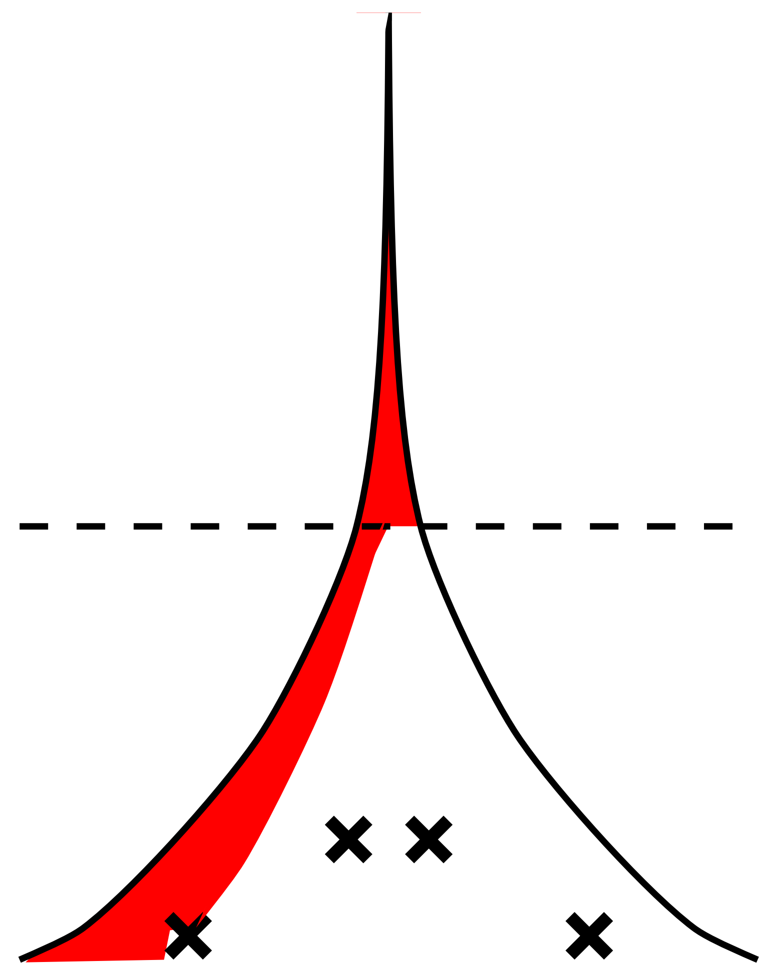
Criterion	Breadth-first	Uniform-cost	Depth-first	Iterative deepening
Complete?	**Yes***	**Yes***	**No**	**Yes**
Time	b^{d+1}	$\approx b^d$	b^m	b^d
Space	b^{d+1}	$\approx b^d$	bm	bd
Optimal?	**Yes***	**Yes**	**No**	**Yes**

Comparison

Breadth-first search



Iterative deepening search



Summary

- **Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored**
- **Variety of uninformed search strategies**
- **Iterative deepening search uses only linear space and not much more time than other uninformed algorithms**