

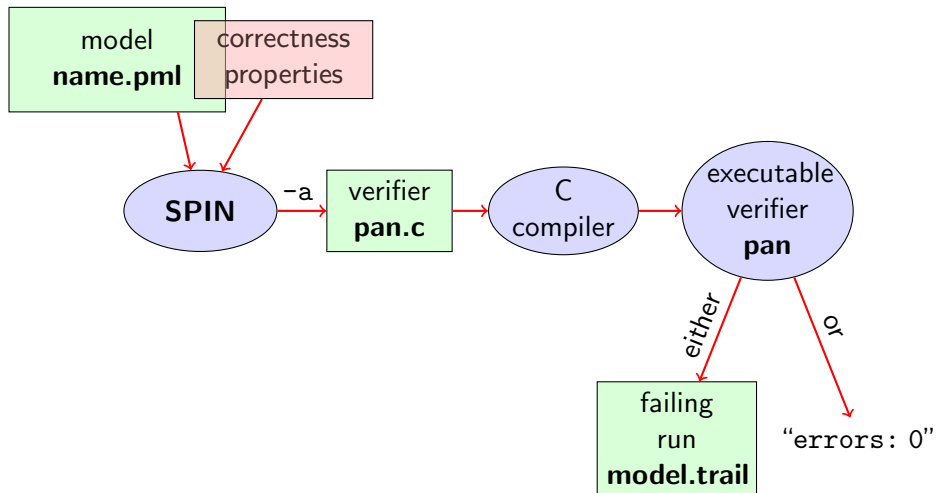
# Formal Specification and Verification

## Model Checking with Temporal Logic

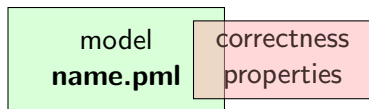
Bernhard Beckert

Based on a lecture by Wolfgang Ahrendt and Reiner Hähnle at  
Chalmers University, Göteborg

# Model Checking with SPIN

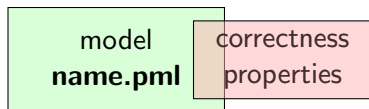


# Stating Correctness Properties



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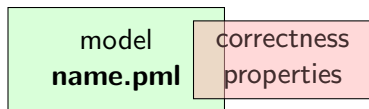


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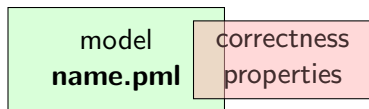


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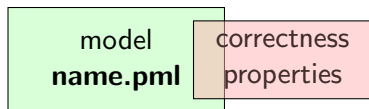
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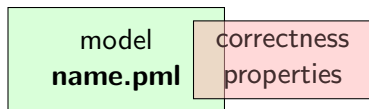
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  - ▶ **accept labels** (briefly)
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- ▶ **temporal logic formulas** (today's main topic)



- ▶ fairness
- ▶ accept labels, accepting runs

# Preliminaries I: Fairness

Does the following PROMELA model necessarily terminate?

```
byte n = 0;
bool flag = false;

active proctype P() {
  do :: flag -> break;
     :: else -> n = 5 - n;
  od
}
active proctype Q() {
  flag = true
}
```

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Termination guaranteed only if scheduling is (weakly) fair!

## Definition (Weak Fairness)

A run is called weakly fair iff the following holds:

each **continuously executable** statement is **executed eventually**.

# Preliminaries II: Acceptance Cycles

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A location marked with an **accept label** of the form “accept<sub>xxx</sub>:” is called an **accept location**.

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Acceptance cycles are mainly used in ‘*never claims*’ (see below), to define forbidden behavior of infinite kind.



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today:

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Remark:

in this course, “temporal logic” is synonymous to “*linear temporal logic*” (LTL)

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- ▶ changing assertions is error prone (easily out of synch)
- ▶ easy to forget assertions:  
correctness property might be violated at unexpected locations
- ▶ many interesting properties not expressible via assertions

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all these are temporal properties  $\Rightarrow$  *use temporal logic*

# Boolean Temporal Logic

talking about numerical variables (like in `critical <= 1` or `0 <= i <= len-1`) requires variation of *propositional temporal logic* which we call **Boolean temporal logic**:

- ▶ **Boolean expressions** (over PROMELA variables), rather than *propositions*, form basic building blocks of the logic

# Boolean Temporal Logic over PROMELA

Set  $For_{BTL}$  of **Boolean Temporal Formulas** (simplified)

- ▶ all PROMELA **variables** and **constants** of type **bool/bit** are  $\in For_{BTL}$

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- ▶ all PROMELA **variables** and **constants** of type **bool/bit** are  $\in For_{BTL}$
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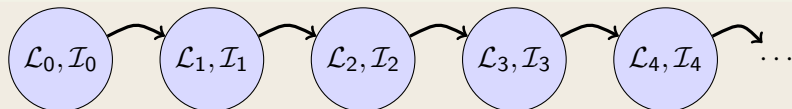
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- ▶ if  $P$  is a process and  $l$  is a label in  $P$ , then  $P@l$  is  $\in For_{BTL}$   
( $P@l$  reads “ $P$  is at  $l$ ”)



# Semantics of Boolean Temporal Logic

A run  $\sigma$  through a PROMELA model  $M$  is a chain of states



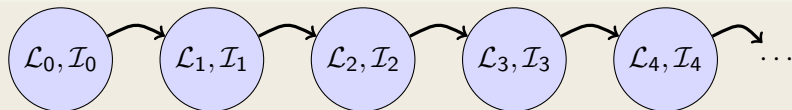
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From  $\mathcal{L}_j$  to  $\mathcal{L}_{j+1}$ , only one of the location counters has advanced  
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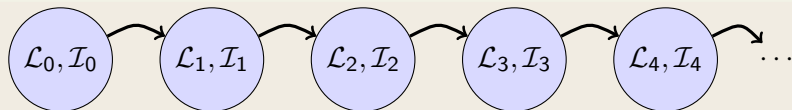
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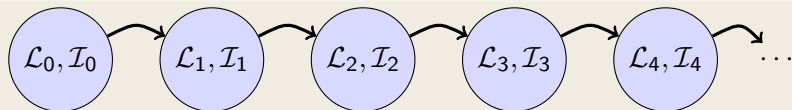
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Evaluating other formulas  $\in For_{BTL}$  in runs  $\sigma$ : see lecture 2.

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instead

Boolean expressions must be **abbreviated** using `#define`

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example:  $[\ ](\text{critical} \leq 1)$

“it is **guaranteed throughout** each run that at most one process visits its critical section”

or equivalently:

“more than one process visiting its critical section will **never happen**”

# Applying Temporal Logic to Critical Section Problem

We want to verify ' $\square(\text{critical} \leq 1)$ ' as correctness property of:

```
active proctype P() {
  do :: /* non-critical activity */
    atomic {
      !inCriticalQ;
      inCriticalP = true
    }
    critical++;
    /* critical activity */
    critical--;
    inCriticalP = false
  od
}

/* similarly for process Q */
```

# Model Checking a Safety Property with JSPIN

1. add `'#define mutex (critical <= 1)'` to PROMELA file
2. open PROMELA file
3. enter `[]mutex` in LTL text field
4. select Translate to create a **'never claim'**, corresponding to the **negation** of the formula
5. ensure Safety is selected
6. select Verify
7. (if necessary) select Stop to terminate too long verification

# Never Claims

you may ignore them, but if you are interested:

- ▶ a never claim tries to show the user wrong
- ▶ it defines, in terms of PROMELA, all **violations** of a wanted correctness property
- ▶ it is semantically equivalent to the **negation** of the wanted correctness property
- ▶ JSPIN adds the negation for you
- ▶ using SPIN directly, you have to add the negation **yourself**

# Model Checking a Safety Property with SPIN directly

## Command Line Execution

*make sure '#define mutex (critical <= 1)' is in safety1.pml*

```
> spin -a -f "!([] mutex)" safety1.pml
```

```
> gcc -DSAFETY -o pan pan.c
```

```
> ./pan
```



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(with  $\text{csp}$  a variable only true in the critical section of  $P$ )

“in each run, process  $P$  visits its critical section **eventually**”

# Applying Temporal Logic to Starvation Problem

We want to **verify** ' $\langle \rangle \text{csp}$ ' as correctness property of:

```
active proctype P() {
  do :: /* non-critical activity */
    atomic {
      !inCriticalQ;
      inCriticalP = true
    }
    csp = true;
    /* critical activity */
    csp = false;
    inCriticalP = false
  od
}

/* similarly for process Q */
/* here using csq */
```

# Model Checking a Liveness Property with JSPIN

1. open PROMELA file
2. enter `<>csp` in LTL text field
3. select Translate to create a 'never claim', corresponding to the negation of the formula
4. ensure that **Acceptance** is selected  
(SPIN will search for *accepting* cycles through the never claim)
5. *for the moment* uncheck Weak Fairness (see discussion below)
6. select Verify

# Verification Fails

Verification fails.

Why?

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Verification fails.

Why?

The liveness property on one process 'had no chance'.  
Not even weak fairness was switched on!



# Model Checking Liveness with Weak Fairness!

Always switch **Weak Fairness** on when checking for liveness!

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3. select Translate to create a 'never claim', corresponding to the negation of the formula
4. ensure that **Acceptance** is selected  
(SPIN will search for *accepting* cycles through the never claim)
5. ensure **Weak Fairness** is checked
6. select Verify

## Command Line Execution

```
> spin -a -f "!csp" liveness1.pml  
> gcc -o pan pan.c  
> ./pan -a -f
```

# Verification Fails

Verification fails again.

Why?

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Why?

Weak fairness is still too weak.

# Verification Fails

Verification fails again.

Why?

Weak fairness is still too weak.

Note that `!inCriticalQ` is **not** continuously executable!

# Temporal MC Without Ghost Variables

We want to **verify mutual exclusion** without using ghost variables

```
#define mutex !(P@cs && Q@cs)

bool inCriticalP = false, inCriticalQ = false;

active proctype P() {
  do :: atomic {
    !inCriticalQ;
    inCriticalP = true
  }
cs: /* critical activity */
    inCriticalP = false
  od
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/* similarly for process Q */
/* with same label cs: */
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Verify '[]mutex' with JSPIN.

# Literature for this Lecture

**Ben-Ari** Chapter 5