Graph Products

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Dujmović et al. [1] proved the following graph product structure theorem.

Theorem 1 ([1])**.** *For every planar graph G there is a graph H of treewidth at most* 8 *and a path P such that $G \subseteq H \boxtimes P$.*

Treewidth measures how similar a graph is to a tree and is an important parameter in algorithmic and structural graph theory. Graphs with bounded treewidth are considered to be a relatively simple class of graphs, in particular a connected graph has treewidth at most 1 if and only if it is a tree. For graphs G and H, the *strong product* $G \boxtimes H$ is the graph with vertex-set $V(G) \times V(H)$ with an edge between two vertices (v, w) and (v', w') if $v = v'$ and $ww' \in E(H)$, if $w = w'$ and $vv' \in E(G)$, or if $vv' \in E(G)$ and $ww' \in E(H)$.

Based on Theorem 1, more graph product structure theorems have been developed, all of which proved useful to resolve longstanding open problems regarding queue layouts, different kinds of coloring problems, adjacency labelings, twin-width, and many more. The project aims at continuing the research in this direction by investigating how well different graph parameters behave under the strong product or other graph products. Possible research directions include

- Finding graph parameters f such that if $f(G) \leq k$ for some graph G, then $f(G \boxtimes P)$ is bounded by a function of k, where P is a path.
- Finding graph parameters such that the above is not true, possibly even if G is planar.
- Replace the strong product by other graph products.
- Replace the path by other graphs.

References

[1] Vida Dujmović, Gwenaël Joret, Piotr Micek, Pat Morin, Torsten Ueckerdt, and David R. Wood. "Planar graphs have bounded queue-number." In: *J. ACM* 67.4 (2020), Art. 22, 38. ISSN: 0004-5411. DOI: 10.1145/3385731. URL: https://doi.org/10.1145/3385731.