Relational Verification

16th KeY Symposium
July 2016

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Title refinement

More precisely:

Effective/Efficient Verification of Relational Properties

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  (set of input-/output-values, set of states)

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- Program $P \subseteq E$
- Deterministic Program $P : D \to D, x \mapsto P(x)$
  $P = \{(x, P(x)) \mid x \in D\} \subseteq E$

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Ex.: for $D = \mathbb{Z}$
$Hom_+ \in E^3$

P satisfies Hom_+ iff $P(x+y) = P(x) + P(y)$

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- that operate on x_1 and x_2
- Proof obligation:
  $[P_1][P_2](\ (old(x_1), x_1, old(x_2), x_2) \in R\ )$
- Often:
  $x_1 \sim_{in} x_2 \to [P_1][P_2] x_1 \sim_{out} x_2$

Instances

- **Non-interference (information flow)**

$$\mathit{low}_1 = \mathit{low}_2 \;\rightarrow\; [P_1][P_2]\; \mathit{low}_1 = \mathit{low}_2$$

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- **Relational Algorithmic Properties**, e.g., voting schemes

$$election_1 \sim eletion_2 \rightarrow [P_1][P_2]\, winner_1 \approx winner_2$$

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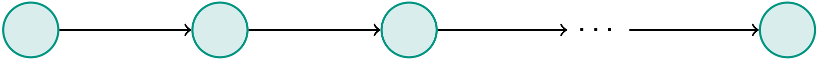
Synchronised Traces



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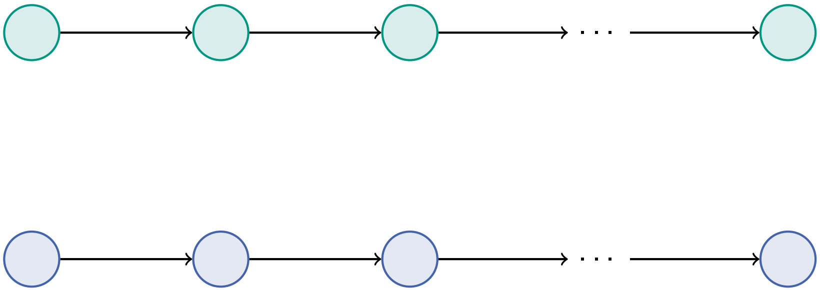

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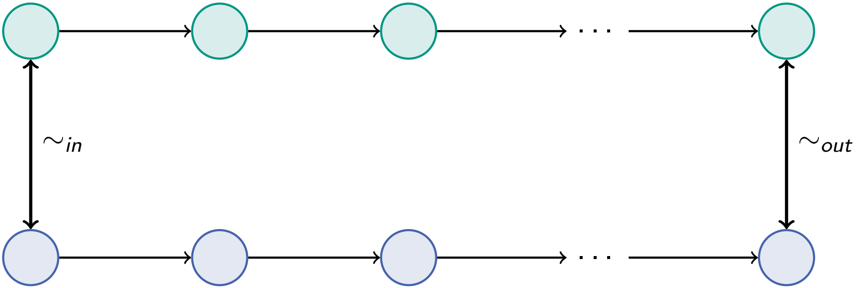

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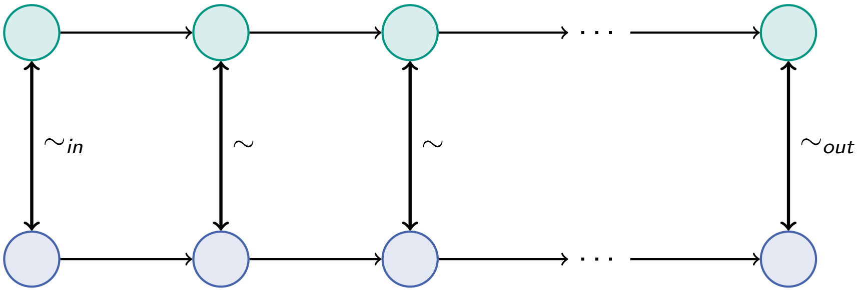

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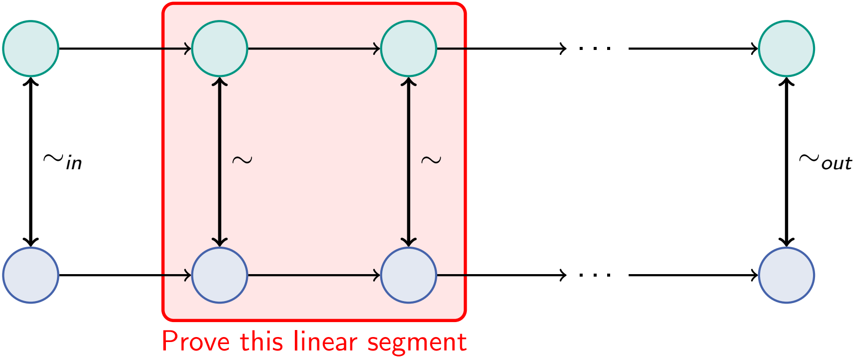

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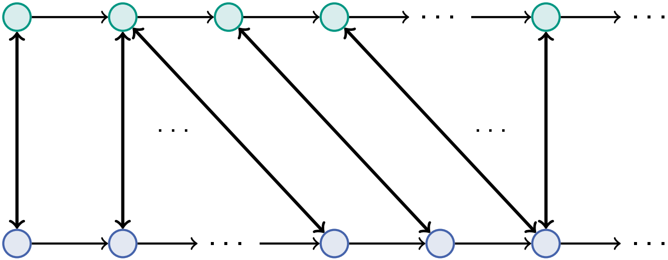

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Justification:

- Strongest loop abstraction is a functional relation.
- Any invariant weaker than the strongest has one input-state x_1, x_2 such that two post-states satisfy it.
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Strongest functional invariants hard to specify/infer.
$\Rightarrow$ Relational regression verification is promising!

Contributions so far

- Refinement from algorithms to implementations [Ulbrich 11]
- Non-interference calculus in KeY [LOPSTR 13]
- Regression verification of C source code [ASE 14]
- Regression verification on PLC code [ICFEM 15]
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Similar techniques are used.
To be effective/efficient, technique must match application.

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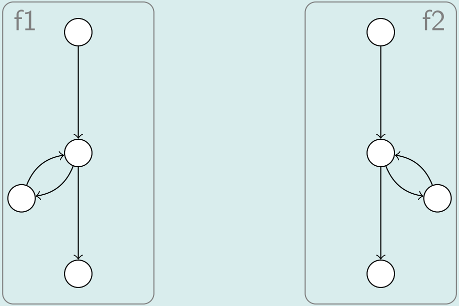
Loop synchronisation



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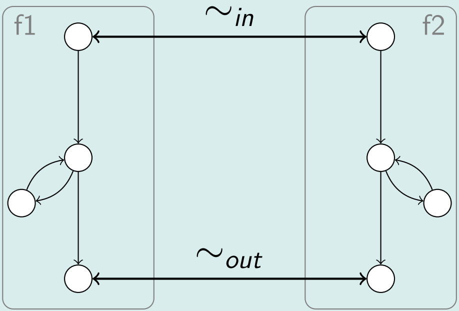

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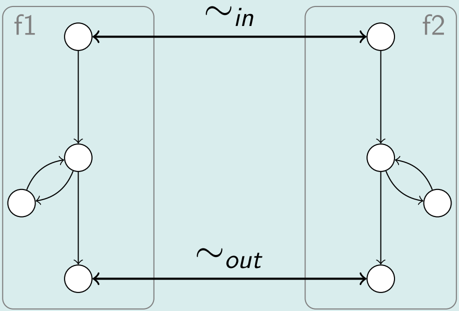

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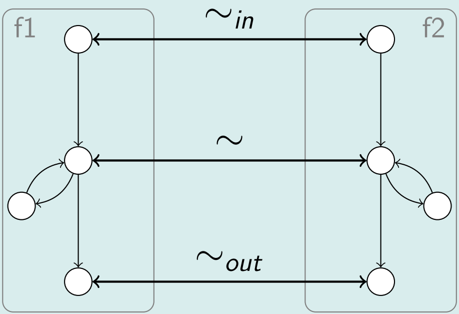

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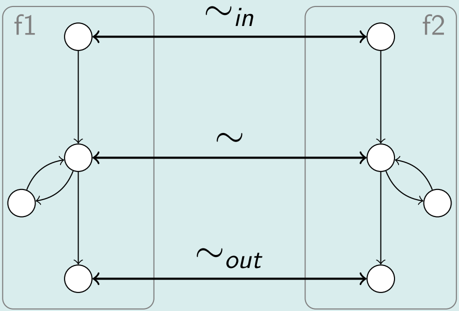

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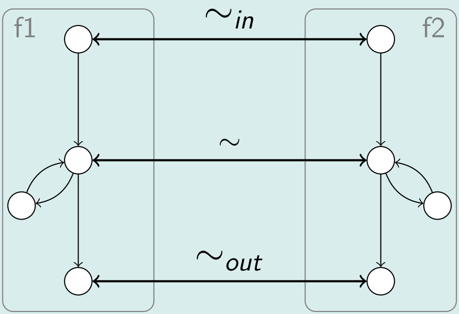

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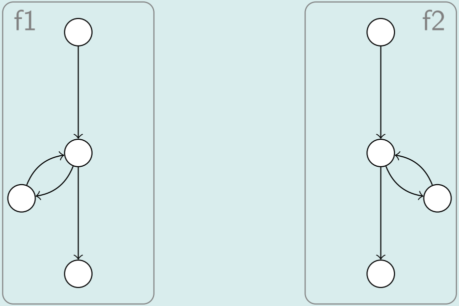

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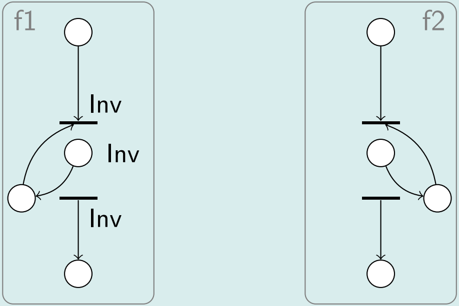

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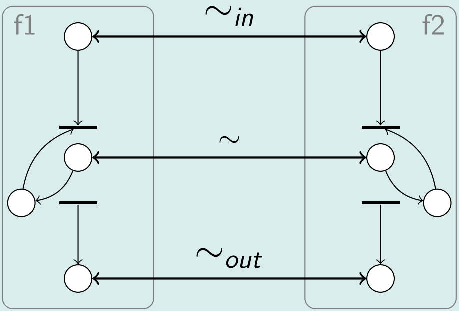

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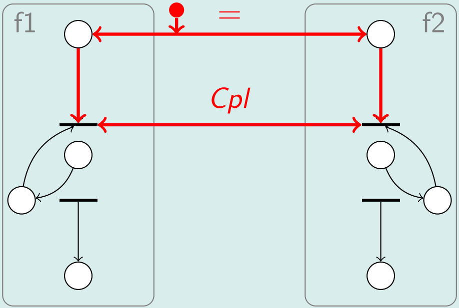

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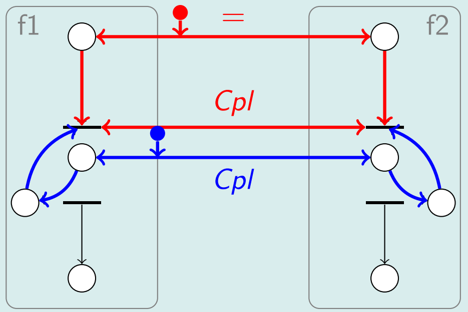

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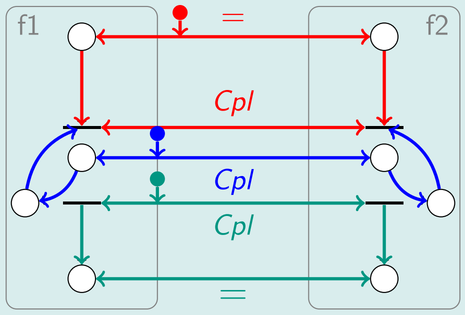

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